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Committing to trade: A theory of intermediation ☆

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ABSTRACT

In a "lemons" market, a shock to gains from trade precedes the buyers' offer. Lower gains exacerbate adverse selection. Trading with intermediaries before observing the shock commits sellers not to keep high-quality assets in such states, improving surplus despite impeding efficient use of information. To add value, intermediaries need not possess superior skills or information. If sellers choose intermediaries to overcome search frictions, traded assets' quality and welfare increase with search costs. The theory offers a novel perspective on the underwrite-to-distribute model in leveraged loans, and predicts that dealers' shift from market-making to match-making may worsen adverse selection in over-the-counter markets.

1. Introduction

Picture yourself negotiating the purchase of a used car with its owner. You present the car owner with hard evidence that the highest price you can afford to pay is low. For instance, market conditions may have made obtaining a car loan unusually costly. The car owner is willing to make a price concession. This leads you to question the car's quality and revise your willingness to pay accordingly. You would likely be less suspicious and require a lower price concession if the owner had based the decision to sell you the car on the higher price they *expected* to extract from you. This would occur in an intermediated market: car dealers approaching owners beforehand would offer them a price that reflects the cars' *average* resale value, motivating a larger number of high-quality owners to sell than in a counterfactual world in which they happened to negotiate with buyers with low willingness to pay.

This paper shows that, in markets plagued by adverse selection, sellers may be better off when their assets are first sold to intermediaries, who sell them to final buyers only after additional information on market participants' private valuations — hence on the gains from trade — is revealed. Notably, the underlying mechanism neither requires intermediaries to possess scarce skills nor imposes any structure on the information they hold about the asset's fundamentals, making the theory applicable to a wide range of scenarios. For instance, the theory offers a novel perspective on the underwrite-to-distribute model in the market for leveraged loans, where banks appear to have abandoned their classic monitoring role (Blickle et al., 2020). In other environments, such as the secondary market for corporate bonds, the fact that dealers intermediate trades likely responds to a demand for immediacy and economizes on search frictions. When I tailor the model to such environments, I show that its core mechanism continues to operate and leads to the striking result that total welfare is increasing in the severity of search frictions. Thus, policy efforts to improve traders' access to counterparties may backfire.[1] Within this application, I endogenize the dual form of intermediation, market-making and matchmaking, generating predictions that link dealers' inventory capacity with the

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[1] The majority of activity in the corporate bond market in the US has shifted almost a century ago from limit-order, to fragmented over-the-counter markets, as documented in Biais and Green (2019). In light of concerns of excessive dealers' market power, some scholars have proposed regulations to promote a shift back to limit-orders through the use of electronic trading platforms (see, for example, Harris et al. 2015).

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severity of adverse selection, and thus contributing to a literature that studies the effects of lowered liquidity provision by dealers (Bao et al., 2018; Kargar et al., 2021; Saar et al., 2023).

Preview of the model and results. The setup is a classic model of adverse selection la Akerlof (1970) with one additional feature: public shocks to gains from trade that realize just before buyers make offers. The addition captures a variety of phenomena. For example, a specialized fund may attract fewer inflows or hit hard industry portfolio constraints, lowering the demand for the assets it most usually trades. In other contexts, the original owners of an asset may be hit by shocks to their cash holdings, affecting their willingness to sell their asset.

Sellers are privately informed about their assets' quality and their private valuation. The gains from trading with buyers depend on the realization of a stochastic variable that I label "the shock". The model has two stages, and the shock realizes and is publicly observed only in the second stage. To simplify the exposition, the baseline model considers the case in which the shock only affects the buyers' valuation for the asset.[2] Therefore, states in which the shock is high (low) are also referred to as high (low) demand states hereafter.

To crystallize the intuition behind the novel mechanism, I begin by comparing two exogenous market setups (Sections 2, 3 and 4). In the *direct*, or *non-intermediated* market, no relevant decision is made in the first stage, and buyers and sellers negotiate directly in the second stage. Competitive buyers bid up to their valuation, reflecting their belief about the asset's quality. In the *intermediated* market, sellers receive offers from competitive intermediaries in the first stage. In the second stage, intermediaries observe the realization of the shock and receive offers from competitive buyers. In both setups, only sellers receive a positive expected payoff, which in aggregate, amounts to total expected surplus.

If intermediaries have low private value for holding the asset, they bid up to its expected resale price. The resulting allocation is the same *as if* sellers in the direct market would commit in the first stage, after observing their types but before observing the demand state, to either retain the asset or accept the best buyers' offer for any realization of the shock.

The fact that sellers, as a result, are better off is not obvious. In the direct market, potentially valuable information on the gains from trade is incorporated in the buyers' bid, whereas it is not available to sellers in an intermediated market. However, due to the lemon's problem, making sellers commit their decision before observing the shock also presents benefits.

Specifically, decoupling the sellers' decision to trade from the realization of the shock can alleviate adverse selection in two ways. First, if *expected* demand is sufficiently high, the intermediaries' offer would also be high: the intermediated market would then generate a larger expected volume of trade of good-quality assets, compared to the direct market. In some cases, this results in larger expected surplus. Second, the intermediated market redistributes trades of good assets from high demand states to low demand states. Compared to the direct market, sellers with relatively low (high) valuation, who are marginal when demand is lower (higher), sell in a larger (smaller) set of states. Crucially, the net surplus induced by the *marginal* trade in the direct market is higher when the shock is lower. The reason is that buyers apply a larger adverse selection discount in these states: since a lower shock reduces the price offer, simple Bayesian reasoning implies that those sellers who wish to trade at that price are more likely to do so because of the asset's low fundamentals rather than for idiosyncratic motives. Thus, the gap between the marginal seller's and the buyer's valuation for the asset is higher.[3] Under conditions, the redistributive effect of intermediation dominates the loss of information that it entails, and expected surplus is higher when trades are intermediated (Theorem 1).

The general insight that intermediation is more valuable when adverse selection is more severe and when gains from trade are more volatile is confirmed by comparative statics results, which contribute to a set of predictions summarized in Section 7.

Until this point, the analysis assumes that intermediaries have exclusive access to buyers. Because exclusivity increases total surplus, it would obtain if trading relationships could be formed and contracts be signed before sellers' types are realized. In Section 5, I explore the case in which exclusivity cannot be enforced. To support equilibria with intermediated trades, one must introduce a benefit for sellers to sell to trade with the intermediary. Out of a several forms of value add, one natural candidate, inspired by a large literature on over-the-counter (OTC) markets, is that search frictions make it costly to reach out to buyers, with sellers facing larger search costs than intermediaries.[4] I first show that, independently of the degree of search frictions, adding intermediaries reduces adverse selection and thus increases the expected volume of trade and expected welfare. I then characterize the equilibrium in presence of intermediaries under any degree of search frictions. By studying the welfare properties of the equilibrium when search frictions are moderate and both intermediated and direct trades occur, I obtain the second main result of the paper. There, a marginal decrease in search frictions leads to disintermediation — that is some sellers switch from selling to the intermediary to directly reaching out to buyers — but also induces sellers with relatively higher valuation to search instead of retaining the asset. Because low valuation sellers are marginal in states in which demand is low and adverse selection is high, the same conditions that make intermediation valuable (as in Theorem 1) yield the paradoxical effect that the decrease in search friction reduces traded assets' average quality and total welfare net of search costs (Theorem 2). The opposite occurs in a market in which intermediaries are not available.

The dual market structure above may also represent a dealer-intermediated market in which sellers can choose between agency trades and principal trades (discussed in Section 8). Within such application, I show that an increase in dealers' inventory costs leads to a shift from principal to agency trades, reducing trade volume and welfare beyond its direct effect. The mechanism helps interpret large illiquidity events in dealer-intermediated markets, a phenomenon at the center of recent research (Yang and Zeng, 2021).

In Section 6, I introduce more general contracts endowing intermediaries with only partial discretion on which offers to accept. I show conditions under which the surplus-maximizing contract takes a simple partitional form: intermediaries must accept the buyers' offer in states in which the shock is higher than a threshold; otherwise, sellers have discretion, and those with good assets reject the offer. Threshold delegation contracts also emerge in the organizational economics literature, but in this paper they serve a novel purpose[5]: they limit downside risk to good-quality sellers, inducing more of them to participate, and in turn, alleviating the adverse selection problem in those states in which the asset is sold.

[2] In a more general formulation, the shock can affect both the buyers' and the sellers' valuations (see Appendix A). In the general setup, all the model's intuitions carry through. The setup nests the case where the shock only affects sellers' valuations. In that case, perhaps the best interpretation is that the shock represents the unpredictable component of sellers' liquidity needs.

[3] Observe that the same logic would apply in a context in which the shock entered the sellers' valuation. When all sellers' valuation for an asset is higher, rational buyers infer that, if an asset is traded, its fundamentals are more likely to be low. For a theoretical reference, this idea is at the core of the results in Malherbe (2014), where sellers who hoard more cash ex-ante are perceived to be more likely to sell assets of bad quality later on.

[4] The assumption can also be viewed as a metaphor for sellers' demand for immediacy. In many search-based models, sellers are hit by negative preference shocks which decrease the flow payoff that they receive from holding the asset.

[5] See, for example, Alonso and Matouschek (2007).

Contribution to the literature. My analysis is closest to the information-based theories of intermediation by Glode and Opp (2016) and Jovanovic and Menkveld (2024). The core inefficiencies and the intermediaries' role in alleviating them distinguish my setup from theirs. In Glode and Opp (2016), uninformed sellers charge an inefficiently high price to screen buyers. Involving moderately informed intermediaries makes it less tempting to do so.[6] In Jovanovic and Menkveld (2024), limit orders put sellers under the risk of news arrival on the asset's common value that are privately observed by buyers. High-frequency traders alleviate this problem by refreshing quotes as new information arrives. An important distinction is that the mechanism in my model operates *irrespective* of the intermediaries' information on the asset's fundamentals. Similarly, the paper departs from Alti and Cohn (2022), in which intermediaries must observe the asset's quality to credibly signal it to buyers through the acquisition price.

In models of intermediation based on search frictions, intermediaries typically add value by providing immediacy (Rubinstein and Wolinsky, 1987; Duffie et al., 2005; Farboodi et al., 2023) or more efficient matching (Chang and Zhang, 2021). Notably, within a search framework, Farboodi et al. (2025) and Bethune et al. (2022) find that more skilled agents (i.e. those with higher bargaining skills or screening ability, respectively) become the core of the trading network, but their presence does not necessarily add value. My contribution to this strand of the literature is twofold: first, when I introduce costly search, I offer an environment in which intermediaries improve welfare over and above the direct immediacy benefits they provide. Second, the analysis reveals a causal link between intermediaries' inventory capacity and the severity of adverse selection. By contrast, the papers above mostly abstract from imperfect information on the asset's fundamentals.

The extension with search frictions connects my paper to the literature incorporating asymmetric information in search models of OTC markets (Weill, 2020). In Chiu and Koeppl (2016) search frictions can exacerbate adverse selection. In my model, when (and only when) intermediaries are present, search frictions may alleviate adverse selection and increase welfare.

The paper contributes more generally to the literature on adverse selection in decentralized asset markets.[7] Trade volume fluctuates endogenously in Asriyan et al. (2019) and Maurin (2022). My model is multi-period but static, and features exogenous shocks affecting trade volume. These shocks are crucial to introduce a potential role of intermediation, a question that does not pertain to the papers above.

Several other works focus on the interplay between market structure and information in financial markets. Notable examples include Duffie et al. (2017) and Glode and Opp (2020). A key element in both works is that information on gains from trade improves efficiency. In my model, information about the shock exacerbates adverse selection, introducing a role for intermediaries: they structure the market so that sellers make decisions before such information realizes or becomes observable.

Roadmap. Section 2 describes the model. Section 3 characterizes equilibria. Section 4 compares surplus across the two market setups and derives comparative statics. Section 5 presents an environment with search frictions. Section 6 extends the contract space. Section 7 discusses the model's predictions. Section 8 provides details on two applications. Extensions, alternative specifications and formal proofs are in Appendices A and B.

[6] In Glode et al. (2019), this intuition is used to prove that a private information structure along the intermediation chain exists such that, when the chain is sufficiently long, the market equilibrium is efficient.

[7] The literature includes, among many others, Chang (2018), Guerrieri and Shimer (2018), Williams (2021) and Fuchs et al. (2022).

2. The model

2.1. Agents and assets

The economy lasts two periods, indexed by $t = 0, 1$. Agents live in both periods and derive utility from holding the asset at the end of $t = 1$, in a way specified below. If they sell (buy) an asset at price p, they receive an additional payoff of p $(-p)$.

Sellers and assets. There is a continuum of sellers, in measure normalized to one, each owning one indivisible asset. Seller differ in a two-dimensional type (θ, x) where $\theta \in \{b, g\}$ describes the quality of the asset they own, and $x \in [0, 1]$ disciplines their utility from holding it at the end of $t = 1$. A share $\lambda \in (0, 1)$ of sellers owns assets of good quality $(\theta = g)$, while the residual holds assets of bad quality $(\theta = b)$. If the asset is of bad quality, their utility from holding it at the end of $t = 1$ is zero. If the asset is of good quality, sellers' valuations, denoted u, are distributed according to a measure G that admits a positive, continuous, and continuously differentiable density, denoted g. To simplify the notation, the seller's type x is defined as the percentile of the distribution of private valuations, u. Formally, the seller's valuation is:

$$U^s(\theta, x) = \begin{cases} u(x) & \text{if} \quad \theta = g \\ 0 & \text{otherwise} \end{cases}$$

where $u(x) := G^{-1}(x)$ is the quantile function of G. Given the properties of G, the function $u(x)$ is strictly increasing, continuous, and has continuous first and second derivatives. Being the percentile of a distribution, x is distributed uniformly in $[0, 1]$.

Buyers and the shock. There is a continuum of buyers, in arbitrarily large measure, initially endowed with no assets. Their utility from holding the asset at the end of $t = 1$ is:

$$U^b(\theta, c) = \begin{cases} v + c & \text{if} \quad \theta = g \\ c & \text{otherwise} \end{cases}$$

where $v > 0$ is constant, and $c > 0$ is a random component that is realized at $t = 1$, is common to all buyers, and drawn from a distribution F defined over the interval $[\underline{c}, \overline{c}]$. For simplicity, c enters buyers' valuation independently on the asset's quality. I hereafter refer to c as "the *shock*". I summarize below the parameters restrictions.

Assumption 1. $(i)\, v > 0$, $(ii)\, u \in C^2$ on $[0, 1]$, $u' > 0$, $(iii)\, c \sim F$ in $[\underline{c}, \overline{c}] \subseteq \mathbb{R}^+$.

Intermediaries. There is a continuum of intermediaries, in arbitrarily large measure, initially endowed with no assets. The value that intermediaries attach to holding the assets at the end of $t = 1$ is:

$$U^i = \begin{cases} u^i & \text{if} \quad \theta = g \\ 0 & \text{otherwise} \end{cases}.$$

In the main text, I assume that intermediaries have no private value for holding the good asset:

Assumption 2. $u^i = 0$.

Section 2.3 discusses the relevance of this assumption and the model's robustness to it.

Information. Sellers can observe their type (θ, x), whereas buyers cannot. Until Assumption 2 is relaxed (Appendix A.1), it is irrelevant whether intermediaries can observe the sellers' types (θ, x). For expositional purposes, I assume they can do so. The assumption is not essential to generate the main results (see Appendix A.2). Finally, the shock c, once realized, is observed by all agents in the economy.

2.2. Trading

I will separately study the two following alternative setups.

Non-intermediated or direct market. At $t = 0$, no relevant decision is made. At $t = 1$, each seller is matched randomly to at least two buyers, each making a binding price offer. The seller observes the offers and chooses whether to sell or retain.

Intermediated market. At $t = 0$, each seller is matched randomly to at least two intermediaries, each making a binding price offer. The seller observes the offer and chooses whether to sell or retain until the end of $t = 1$. Those intermediaries holding assets at $t = 1$ are matched to at least two buyers each, who make binding price offers. Intermediaries observe the offers and choose whether to sell or retain.

2.3. Discussion of the main assumptions

No direct trades in the intermediated market. In the intermediated market described above, sellers cannot reject the intermediaries' offer and trade directly with buyers at $t = 1$. In Section 4.3, I argue that when the intermediated market produces more surplus than the direct market, agents will have incentives to enter agreements that shape the market structure accordingly. In Section 5, I assume that such exclusivity agreements cannot be enforced and add the friction that sellers find it costly to search for buyers (or to delay trade). Within this extension, I characterize equilibria in which some trades are intermediated and some are direct, and obtain several novel insights.

Timing of the shock and interpretation. The central distinction between the two market setups is that only in the direct market sellers observe c when deciding whether or not the trade their assets. In the current setup, this occurs because c realizes at $t = 1$. An equally valid alternative that might fit some applications would be to assume that c is already realized at $t = 0$, but is only observable upon matching to a buyer (see Chang and Zhang 2021). In this alternative too, a chain of trades involving an intermediary would effectively decouple the sellers' decision to trade from the observation of the shock and hence the buyers' price offer.

Buyers' and sellers' preferences. As the analysis will reveal, the mechanism of the paper operates as long as the shock c moves the *net* gains from trading the asset. In a more general specification, the shock c may also enter the sellers' valuation (see Appendix A.3).

Intermediaries preferences. The baseline model assumes the most parsimonious case in which the intermediaries' valuation for the asset is low (and set to zero for simplicity). The restriction may reflect intermediaries' higher risk-aversion or a motive to free up their capacity and profit from other trades.[8] In Appendix A.1, I characterize an upper bound on u^i below which the analysis in the main text remains entirely valid. The upper bound can be large: in an example, it exceeds the sellers' median valuation.[9] Restricting attention to a sufficiently low valuation crystallizes the core intuition of the paper. In Appendix A.1, I find conditions under which the intermediated market dominates the direct market for *any value of* u^i.[10]

[8] For an example where this motive is modeled explicitly, see Jovanovic and Szentes (2013), where venture capitalists are endogenously more eager to terminate projects, i.e. to sell the startup via an IPO, compared to the original entrepreneurs.

[9] Moreover, in the specification in which the shock c only affects the sellers' preferences (Appendix A.3), the upper bound on u^i can still be above the sellers' valuations in states in which they are low.

[10] For intermediate values of u^i, new subtle effects play out and contribute to both the benefits and costs of intermediation. Obviously, above a certain point, increasing the valuation of intermediaries creates gains from trade that mechanically make the intermediaries valuable.

Intermediaries' information over the shock. In Appendix A.2 I show that if intermediaries observed a common imperfect signal of c, the main results would continue to hold in the baseline model. Notably, in the model in which the shock only affects the sellers' valuation, all results would continue to hold irrespective of intermediaries' information.

Contracts. Assuming that, instead of a fixed price, intermediaries could offer a price that depends on the realized c would not alter any of the results—how price risk is shared is irrelevant, unlike for example in Baldauf et al. (2022) and (2024). In a more interesting variation, contracts may give sellers the opportunity to retain the asset ex-post for some realizations of c. A thorough analysis is in Section 6.

Trading protocol. The assumption that uninformed buyers make the offer is made to avoid a formal discussion of signaling considerations. In an alternative model where informed sellers make the offer but can only post a price, the equilibrium (pooling) price would also hold competitive buyers at their participation constraint and all results would continue to hold.

2.4. Equilibrium notion

Under both market setups, at each stage, the buying side of the market breaks-even in expectation. Competitive models of trade under asymmetric information may admit multiple prices that make buyers break-even, given the sellers' optimal response. When this is the case, I select the equilibrium featuring the highest such price. A restriction on buyers' deviating profits that leads to the same refinement can be found, for example, in Asriyan et al. (2019).

Formally, the sellers' strategy profile can be represented by the set of types $\Gamma^s(p)$ who accept the best offer — coming from intermediaries or buyers, depending on the setup — for any best offer, p:

$$\Gamma^s(p) := \{(\theta, x) : p \geq U^s\}.$$

Similarly, the intermediaries' decision at $t = 1$ when confronted with a best offer p can be represented by the set:

$$\Gamma^i(p) := \left\{\theta : p \geq U^i\right\}.$$

When making offers in the direct market, buyers use their prior knowledge of the distribution of (θ, x) and update it with the information embedded in the sellers' response $\Gamma^s(p)$.

In the intermediated market, buyers begin with a prior and form a belief β about the share of intermediaries holding good-quality assets at $t = 1$. For a given offer p, the belief is updated with the information embedded in the intermediaries' response $\Gamma^i(p)$. Finally, $V^i(\theta; \beta)$ denotes the continuation payoff of an intermediary after having bought an asset of quality θ at $t = 0$.

Definition 1 (*Equilibrium*)**.** In equilibrium, sellers' and intermediaries' strategies when receiving offers on and off-equilibrium path are described by the sets $\Gamma^s(p)$ and $\Gamma^i(p)$ for all p. In the direct market, the price at which buyers buy is:

$$p = \sup\left\{q \in \mathbb{R} : q = \mathbb{E}\left[U^b | (\theta, x) \in \Gamma^s\right]\right\}.$$

In the intermediated market, the price at which the intermediaries buy assets of quality θ, denoted $p^i(\theta)$ and the price at which the buyers buy the asset, denoted p^b are:

$$p^i(\theta) = \sup\left\{q \in \mathbb{R} : q = \mathbb{E}\left[V^i(\theta; \beta)\right]\right\}$$

and

$$p^b = \sup\left\{q \in \mathbb{R} : q = \mathbb{E}\left[U^b | \theta \in \Gamma^i\right]\right\}$$

where the expectation is taken using the belief β, and such belief is consistent with equilibrium trading at $t = 0$.

3. Analysis

The two benchmarks below highlight how the two main frictions — unobservability of the shock at $t = 0$ and asymmetric information on the asset's fundamentals — are necessary to make intermediation valuable. Without the first friction, the distinction between the two markets would be irrelevant. Without the second friction, the direct market would be efficient and would generally dominate the intermediated market.

3.1. Benchmark I: Publicly observable shock at $t = 0$

Assume first that the realization of c was publicly known at $t = 0$. Recall that $u^i = 0$ and thus in the intermediated market, competitive intermediaries would offer for both types of asset the same price, reflecting its expected resale value. Since c is not uncertain, the asset's resale value is also deterministic at $t = 0$. It follows that the intermediated market would replicate the outcome of the non-intermediated setup. Clearly, the same argument would apply if one assumed that c was constant in the first place.

Remark 1. When the realization of the shock is observable at $t = 0$, sellers' payoff and aggregate surplus are the same across the two market setups.

3.2. Benchmark II: Publicly observable asset's quality

Assume now that buyers can observe the asset's quality. In the direct market, buyers will bid up to their valuation for each asset. As a result, and since the sellers' utility for keeping an asset of bad quality is zero, all assets of bad quality will be traded. Good quality assets are traded if and only if $v + c \geq u(x)$, hence the market outcome is efficient for every realization of c.

In the intermediated market, intermediaries anticipate at $t = 0$ that bad assets can be sold at $t = 1$ at price c, while good assets can be sold at $t = 1$ at price $v + c$. Since they bid up to each asset's respective expected resale price, every asset of bad quality is sold at $t = 0$, whereas good quality assets are sold if and only if $v + \mathbb{E}c \geq u(x)$. Price offers at $t = 0$ do not incorporate information that will only be available at $t = 1$, leading to either too much or too little trade compared to the efficient benchmark, depending on how large $\mathbb{E}c$ is.

Remark 2. When the asset's quality is publicly observable, the intermediated market generates weakly lower expected aggregate surplus than the direct market.

Whenever information over the realization of c is valuable to determine which agent is the most efficient user of the asset, the intermediated market setup would lead to an inefficient allocation, and thus to lower aggregate surplus than the non-intermediated alternative.

3.3. Analysis of the full model

3.3.1. Direct or non-intermediated market

Recall that no relevant decision is made at $t = 0$, and fix a realization of c at $t = 1$. Sellers sell the good asset at price p if and only if $p \geq u(x)$. Thus, since $u(x)$ is increasing, the sellers' response is described by a cutoff $\hat{x}$, identifying the marginal seller of a good asset. When the marginal seller is interior, type $\hat{x}$ is such that the utility from holding the asset equals the price. If no good asset owner wants to sell, $\hat{x} = 0$. If all of them want to sell, $\hat{x} = 1$. The fact that sellers with bad assets always choose to sell and that buyers must break even gives:

$$p = \frac{\hat{x}\lambda}{\hat{x}\lambda + 1 - \lambda} v + c. \tag{1}$$

It is useful for notational convenience to define the function:

$$\vartheta(x) := \frac{x\lambda}{x\lambda + 1 - \lambda} v.$$

In words, $\vartheta(x)$ indicates, given a marginal seller x, the expected common value component of the buyers' utility for holding the asset, conditional on the information available to buyers, that is, that the asset is traded in the market. The function $\vartheta(x)$ is an inverse measure of the adverse selection discount at which good assets are traded.

Note from (1) that a larger realization of c increases the equilibrium price and hence the sellers' payoff conditionally on choosing to trade. When interior, the marginal seller satisfies:

$$\vartheta(\hat{x}) + c = u(\hat{x}). \tag{2}$$

Under Definition 1, the equilibrium marginal seller, when interior, must be the highest solution to (2). The next result will be useful to characterize the allocative role that the shock c will have on the market equilibrium.

Lemma 1. ***(Marginal trades and the shock).*** *Whenever an interior solution to* (2) *exists, the highest such solution is a strictly increasing function of c.*

I can now characterize the market equilibrium. I do it under a restriction on u and ϑ that ensures that the marginal seller as a function of c exhibits at most one discontinuity, around which the market for good assets unravels.

Assumption 3 (*Regularity*)**.** The function $u'(x)$ crosses $\vartheta'(x)$ at most once, in which case it crosses it from below.

Assumption 3 is weaker than assuming that $u(x) - \vartheta(x)$ is strictly increasing, which would make the marginal seller continuous in c.

Proposition 1 (*Equilibrium in the Direct Market*)**.** *The equilibrium is unique. All bad assets are traded, while good assets are traded if and only if the seller's type is below a cutoff x^*, which is a unique, (weakly) increasing function of c:*

$$x^*(c) = \begin{cases} 0 & \text{if} \quad c < c_1 \\ \sup\{x \in [0,1] : \vartheta(x) + c = u(x)\} & \text{if} \quad c_1 \leq c < c_2 \\ 1 & \text{if} \quad c_2 \leq c. \end{cases}$$

The function $x^*(c)$ has at most one discontinuity, at $c = c_1$. The price at which all assets trade satisfies condition (1) *with $\hat{x} = x^*(c)$.*

The full characterization of the thresholds c_1 and c_2 is in Appendix A.

3.3.2. Intermediated market

To solve for the equilibrium in the intermediated market, note that since intermediaries attach no value for holding either asset, they will sell it for any non-negative offer p^b.

Lemma 2 (*Trading at $t = 1$*)**.** *Given a strategy profile at $t = 0$, the equilibrium in the trading subgame at $t = 1$ is unique. Intermediaries always trade.*

Consider now the trading game at $t = 0$. Intermediaries are competitive and attach to the asset of either quality only its expected resale value. Thus their offer is

$$p^i = \mathbb{E}\left(p^b\right). \tag{3}$$

The expectation in (3) is taken over the possible realizations of c. In turn, sellers of good asset sell at p^i if and only if: $p^i \geq u(x)$. Once again, sellers of good assets sell whenever their type is below a cutoff. Buyers form a belief β about the share of intermediaries holding assets of good quality at $t = 1$, that must be consistent with trading at $t = 0$. Thus, if the marginal seller is of type $\hat{x}$, the belief is

$$\beta = \frac{\hat{x}\lambda}{\hat{x}\lambda + 1 - \lambda}$$

and competition among buyers implies that $p^b = \vartheta(\hat{x}) + c$. Using $p^i = \mathbb{E}\left(p^b\right)$ and Definition 1, the next result follows.

Proposition 2 (*Equilibrium in the Intermediated Market*)**.** *The equilibrium is unique, and assets are either allocated to buyers or retained by sellers. All bad assets are traded, while good assets are traded if and only if the seller's type is below a cutoff X^* that does not depend on c. The cutoff is a function of the prior expectation $\mathbb{E}c$:*

$$X^*(\mathbb{E}c) = \begin{cases} 0 & \text{if} \quad \mathbb{E}c < c_1 \\ \sup\{x \in [0,1] : \vartheta(x) + \mathbb{E}c = u(x)\} & \text{if} \quad c_1 \le \mathbb{E}c < c_2 \\ 1 & \text{if} \quad c_2 \le \mathbb{E}c \end{cases}$$

where c_1 and c_2 are the same thresholds identified in Proposition 1, and $X^*(\mathbb{E}c)$ has at most one discontinuity, at $c = c_1$. The price at $t = 1$ satisfies condition (1) with $\hat{x} = X^*(\mathbb{E}c)$.

The immediate implication of Proposition 2 is that

$$X^* = x^*(\mathbb{E}c). \tag{4}$$

Condition (4) is key to compare the assets' allocation across the two market setups. The remark below follows from it.

Remark 3 (*Retain or commit to trade*)**.** For every c, the allocation in the intermediated market corresponds to that of the direct market when $c = \mathbb{E}c$. The intermediated market induces the same allocation (and the same expected payoffs) that would result from a version of the direct market in which sellers could only choose at $t = 0$, after learning their types but before observing the buyers' offers, whether to retain the asset or accept the best offer for all realizations of c. In this way, those sellers who choose to trade are committed not to retain their assets ex-post.

The statement above provides a useful interpretation of my theory of intermediation: the intermediary's role is to structure the market so that sellers, who maintain private information on their types, can commit not to act on the realized buyers' price when deciding whether to trade the asset, but only on its expectation, based on their prior over c.[11]

The next section establishes conditions under which such commitment increases sellers' expected payoff, and hence aggregate surplus.

4. Main results

4.1. Comparing surplus in the two market setups

Denote with $S_D(c)$ and $S_I(c)$ aggregate surplus under the direct and intermediated market respectively, given a realization of c. Using the result described by condition (4):

$$\mathbb{E}S_I(c) = S_D(\mathbb{E}c). \tag{5}$$

In words, the expected surplus in the intermediated market is the same as in a direct market when the realization of c equals its mean. The result implies:

$$\mathbb{E}S_I(c) > \mathbb{E}S_D(c) \Longleftrightarrow S_D(\mathbb{E}c) > \mathbb{E}S_D. \tag{6}$$

The statement in (6) reveals that comparing expected aggregate surplus across the two market setups boils down to an application of Jensen's Inequality. In particular, the direct market generates higher (lower) expected surplus than the intermediated market if the function $S_D(c)$ is convex (concave) in c. To put it differently, the direct market dominates if making sellers act upon the information embedded in the buyers' price is beneficial, whereas the intermediated market dominates if it is detrimental. As revealed with Benchmark II, the former case applies when asset quality is observable. The main result of this section establishes conditions under which, with unobservable asset quality, the surplus ranking is reversed.

To derive less stringent conditions, I use the fact that, since the buyers' best price offer is at least c and at most $\lambda v + c$, the marginal seller, $x^*(c)$, is bounded below by $\underline{x}(c) := \max\{0, u^{-1}(c)\}$ and above by $\bar{x}(c) := \min\{1, u^{-1}(\lambda v + c)\}$. Since $x^*(c)$ is non-decreasing, the marginal seller must lie in the set $\chi := [\underline{x}(\underline{c}), \bar{x}(\bar{c})]$.

Theorem 1 (*Surplus Comparison*)**.** *In either of the two cases below, the intermediated market dominates (i.e., $\mathbb{E}S_I(c) \ge \mathbb{E}S_D(c)$):*

(I). $\mathbb{E}c \ge c_2$ and $\mathbb{E}\left[\int_{\bar{x}(c)}^1 (v + c - u(x))\,dx\right] \ge 0$.

(II). For all $x \in \chi$

$$\left[u''(x) - \vartheta''(x)\right]\left[v - \vartheta(x)\right] \ge \left[u'(x) - 2\vartheta'(x)\right]\left[u'(x) - \vartheta'(x)\right] \tag{$\star$}$$

and either

(i). $\underline{c} \ge c_1$ (no unraveling) or

(ii).

$$\int_0^{\underline{x}(\mathbb{E}c)} v + \underline{c} - u(x)\,dx \ge \frac{\left(\mathbb{E}c - \underline{c}\right)\left(v - \vartheta\left(\underline{x}(\mathbb{E}c)\right)\right)}{u'\left(\underline{x}(\mathbb{E}c)\right) - \vartheta'\left(\underline{x}(\mathbb{E}c)\right)} \tag{$\star\star$}$$

A sufficient condition for ($\star$) to hold is that for all $u \in \left[u\left(\underline{x}\left(\underline{c}\right)\right), u\left(\bar{x}\left(\bar{c}\right)\right)\right]$:

$$\frac{g'(u)}{g(u)} \le -\left[(1-\lambda)v\right]^{-1}$$

At the end of this section, I describe examples of specifications in which the conditions in Theorem 1 are satisfied.

Intuition for Part I (when intermediation increases the volume of trade). Part (I) reveals a direct way in which the intermediated market structure may result in higher surplus. When the expected shock is sufficiently high ($\mathbb{E}c \ge c_2$), all sellers accept the intermediaries' offer, hence all assets are ultimately allocated to buyers. This allocation improves over the direct market outcome provided that, when averaging across all realizations of c, sellers above the marginal type $x^*(c)$ value the asset less than buyers. Note that the condition is implied by the assumption of "universal" gains from trade ($v + c \ge u(x)$ for all c and all x).

Intuition for Part II (when intermediation redistributes trade across states). Assume now that $X^* < 1$: some sellers reject the intermediary's offer. In this case the intermediated solution does not necessarily lead to a greater unconditional volume of good assets traded at $t = 1$. Compared to a direct market, the intermediated one induces a larger volume of trade in states in which demand is low, but a smaller volume of trade in states in which demand is high. If $S_D(c)$ is concave, the redistribution of trade across states is beneficial.

Aggregate surplus in the direct market for a given realization of c is:[12]

$$S_D(c) = (1-\lambda)c + \lambda\left[\int_0^{x^*(c)} (v + c)\,dx + \int_{x^*(c)}^1 u(x)\,dx\right].$$

[11] This is key to understand the contribution of the paper. Indeed, note that making sellers trade before they even learn their type would eliminate the adverse selection problem to start with. This is not what occurs in my model, and is an observation that belongs to an earlier literature (e.g. Bolton et al. 2011). Note that trading before private information would not require intermediation—any mechanism that shifts the timing of trade would do. By contrast, in my model, one could interpret the shock c as being already realized at $t = 0$ but only revealed through direct negotiation with the buyer, making the intermediary essential to create the necessary commitment.

[12] Clearly, $S_I(c)$ is given by the same function when $X^* = x^*(\mathbb{E}c)$ replaces $x^*(c)$ for all c.

Consider first the region $[c_1, c_2]$, where $x^*(c)$, the marginal seller of a good asset, is interior. Computing the first derivative of $S_D(c)$, one gets:

$$S'_D(c) = \underbrace{1 - \lambda + \lambda x^*(c)}_{\text{increase in surplus from inframarginal trades}} + \lambda x^{*'}(c) \underbrace{[v + c - u(x^*(c))]}_{\text{net surplus from marginal trade}}$$

The first component of $S'_D(c)$ represents the marginal increase in surplus generated from every inframarginal trade, as a result of an increase in c. This effect amounts to the total measure of assets that trade at c, which is $1 - \lambda + \lambda x^*(c)$. I label this effect the *inframarginal trades effect*. Since $x^*(c)$ is strictly increasing, the inframarginal trades effect is larger in higher states. Thus, absent other effects, the inframarginal trades effect would make $S_D(c)$ convex: when sellers and buyers trade directly, more (fewer) trades occur when buyers' valuation is higher (lower). This reflects the value of the information embedded in the buyers' price, which sellers can act upon only in a direct market.

The second component of $S'_D(c)$ represents the marginal effect on surplus of an increase in c through its effect on the marginal seller. Using the indifference condition (2), rewrite the net surplus generated by the marginal trade as:

$$v + c - u(x^*(c)) = v - \vartheta(x^*(c)).$$

The fundamental observation to be made is that, due to asymmetric information, the net surplus generated by the marginal trade *decreases* in c. An equal increase in the marginal seller's valuation increases surplus by more when the marginal seller is low to begin with, which occurs in states where c is low. Therefore, this effect, which I label *marginal trades effect*, is a source of concavity of $S_D(c)$.

The *marginal trades effect* is modulated by the endogenous change in $x^*(c)$. To inspect the behavior of $x^*(c)$, recall that the marginal seller at every c is defined implicitly by[13]:

$$\vartheta(x^*(c)) + c = u(x^*(c)). \tag{7}$$

To gain insights onto the restrictions implied by ($\star$), assume first that the quantile function $u(x)$ is linear (i.e. sellers' values are uniformly distributed). Note that $\vartheta(x)$ — the expected common value component of the asset's payoff — is concave. When the measure of good asset holders who trade, x, is small to begin with, as it is the case when c is low, a marginal increase in it produces larger changes in buyers' beliefs. Since these changes translate into larger increases in the price, which is the left-hand side of (7), the valuation of the marginal seller, the right-hand side, must be more sensitive to c in low states, resulting in a concave $x^*(c)$. Concavity of $x^*(c)$ is implied by $u''(x) > \vartheta''(x)$, and thus it is preserved when the concavity of the quantile function $u(x)$ is not too pronounced ($u''(x)$ is negative but small in absolute value) or whenever $u(x)$ is convex. These observations motivate the statement at the end of Theorem 1: if the density g decreases sufficiently fast (equivalently, u''/u' is sufficiently large), the *marginal trades effect* always dominates. In general, the sufficient condition ($\star$) imposes weaker requirements on the distribution of sellers' valuation to guarantee concavity of $S_D(c)$. Fig. 1 provides an illustration.

If the marginal seller is always interior ($\underline{c} \geq c_1$), condition ($\star$) implies concavity of $S_D(c)$ and the intermediated market dominates for every prior distribution of c.

If $\underline{c} < c_1$, there exists states in the direct market in which no good assets are traded: the market unravels. Below c_1, the marginal trades effect ceases to exist, hence the slope of $S_D(c)$ in such a region is below the slope in $[c_1, c_2]$, creating a local convexity.[14] To apply Jensen's inequality given a prior $\mathbb{E}c$ it suffices to prove that the tangent to $S_D(c)$ at $\mathbb{E}c$ lies entirely above S_D. This requirement yields condition ($\star\star$), which ensures that sufficiently large surplus generates from the trades of types in $[0, X^*]$, even at the lowest c. Figs. 2 and 3 depict situations in which, respectively, case (II) and case (I) of Theorem 1 apply.

Below, I present two examples which meet the conditions of Theorem 1.[15] In the first one, the density g decreases sufficiently fast to make ($\star$) hold, but the possibility of market unraveling requires to check ($\star\star$). In the second, g is flat but ($\star$) holds nonetheless.

Example 1. Sellers' valuations are distributed exponentially, with $g(u) = \alpha e^{-\alpha(u-1)}$ for $u \geq 1$, therefore the quantile function is $u(x) = -1/\alpha \ln(1-x) + 1$. To ensure that Assumption 3 holds, impose that the lower bound on u' is above the upper bound on ϑ', giving $\alpha < 1/\lambda v$. If $g' \leq -g/(1-\lambda)v$, then ($\star$) holds. This requires that $\alpha > 1/(1-\lambda)v$. Set $\alpha = 2$, $\lambda = 1/20$ and $v = 2$. Since $u(0) = 1$, the market may unravel for some values of $\underline{c}$. Indeed, if $\underline{c} = 8/10$ verify that $\lambda v + \underline{c} < u(0)$ where the left-hand side provides an upper bound on the equilibrium price when $c = \underline{c}$. For the intermediated market to dominate, one must then check that ($\star\star$) holds. Noting that in this example, $\underline{x}(\mathbb{E}c) = 1 - e^{-\alpha(\mathbb{E}c-1)}$, verify that, at $\mathbb{E}c = 2$, the equilibrium is interior and condition ($\star\star$) is satisfied with strict inequality.[16]

Example 2. Sellers' valuations are uniform in $[\underline{u}, \bar{u}]$, with $g(u) = (\bar{u} - \underline{u})^{-1}$ and $u(x) = (\bar{u} - \underline{u})x + \underline{u}$. Thus, $u' = \bar{u} - \underline{u}$ and $u'' = 0$ for all x, and $g' = 0 > -g/(1-\lambda)v$. Since $u(0) = \underline{u}$, the market for good assets never unravels if one assumes $\underline{c} > \underline{u}$. Inspecting ($\star$), note that, at $x = 1$, one gets the lower (upper) bound on the left-hand (right-hand) side of the inequality. Thus, for ($\star$) to hold it suffices that: $2[\lambda(1-\lambda)v]^2 \geq [\bar{u} - \underline{u} - \lambda(1-\lambda)v][\bar{u} - \underline{u} - 2\lambda(1-\lambda)v]$. Set $\bar{u} = 4$, $\underline{u} = 2$, $v = 3$, and $\lambda = 1/2$ for an example where this is satisfied. Moreover, if $2 < \underline{c} < u(1) - \lambda v = 5/2$, interior equilibria exist for some c.

4.2. Comparative statics. When are intermediaries more valuable?

I now study how changes in the model primitives affect the value of intermediation, defined as

$$\text{value of intermediation} = \mathbb{E}S_I(c) - \mathbb{E}S_D(c).$$

4.2.1. Sellers' and buyers' values

Recall that sellers' valuations are described by $U^s(\theta, x)$ with $U^s(b, x) = 0$ and $U^s(g, x) = u(x)$. I will consider small perturbations to $U^s(\theta, x)$, therefore any sufficiently small change in $U^s(b, x)$ will not affect the sellers' behavior when they hold the bad asset: they will sell it at any price in equilibrium. I thus focus on the function $u(x)$ that disciplines sellers' valuation of the good asset. I consider perturbations of the form:

$$\tilde{u}(x) = u(x) + \varepsilon p(x)$$

for small values of the positive scalar, ε.[17] For buyers' valuations, recall that $U^b(\theta, c)$ denotes the valuation for an asset of quality θ, when the shock equals c. This valuation is homogeneous across buyers. Therefore, I will consider the effect of a uniform shift in $U^b(\theta, c)$.

Proposition 3 (*Changes in Sellers' and Buyers' Values*). *Assume condition ($\star$) holds, $\underline{c} \geq c_1$ (no unraveling), $u''(x) > \vartheta''(x)$, and $u'''(x) < \vartheta'''(x)$ for all $x \in \mathcal{X}$. Then:*

---

[13] By differentiating (7), one gets $x^{*'}(c) = [u'(x^*(c)) - \vartheta'(x^*(c))]^{-1}$. From there, simple calculations lead to condition ($\star$), which is in fact equivalent to assume that the expression: $x + \frac{v - \vartheta(x)}{u'(x) - \vartheta'(x)}$ is decreasing in x.

[14] Moreover, the function is generically discontinuous at c_1 ($x^*(c_1) > 0$).

[15] Two other examples are in Appendix A.

[16] Observe that, for any y, $\int_0^y [-1/2 \ln(1-x) + 1]\, dx < -1/2 y \ln(1-y) + y$, which can be used to identify a lower bound on the left-hand side of ($\star\star$).

[17] Technically, I take the variational (Gateaux) derivative of endogenous variables of interest such as, for example, the indifferent seller x^* with respect to changes in the function u in the direction of p. Details are in Appendix A. See the recent Dávila and Walther (2023) for an example of a fruitful use of this methodology in a finance application.

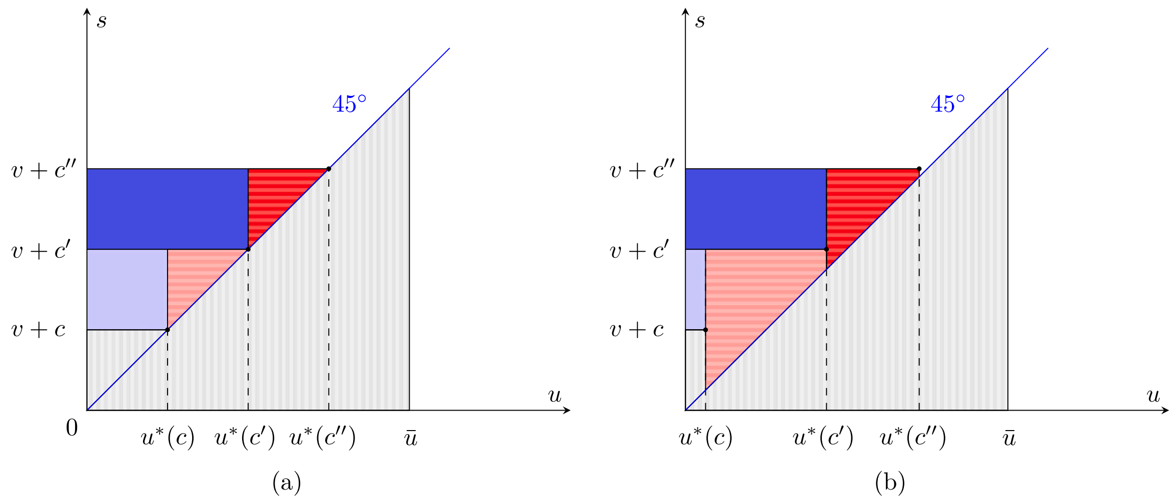

Fig. 1. Figure (a): on the horizontal axis is the good asset holder's valuation and $u^*(c)$ indicates the valuation of the marginal seller, that is type $x^*(c)$. On the vertical axis is the surplus produced by the good asset owner. If the asset is sold, this is $v + c$, otherwise, this is u. With uniformly distributed valuations, the light-gray (vertical stripes) area gives aggregate surplus produced by good asset holders at c. Since $v + c = u^*(c)$, the picture illustrates the benchmark where assets' quality is observable. The light-blue (light-gray, no stripes) area and the light-red area (light-gray, horizontal stripes) are, respectively, the ***inframarginal trades effect*** and the ***marginal trades effect*** for an increment from c to c'. For small increments, the marginal trades effect vanishes. The dark-blue (dark-gray, no stripes) and dark-red (dark-gray, horizontal stripes) areas are the two effects when the shock increases by the same amount from c' to c^{ε}. The inframarginal trades effect is higher, while the marginal trades effect remains constant: $S_D(c)$ is convex.Figure (b): here, $v + c > u^*(c)$, thus this is an environment with unobservable assets' quality. The inframarginal trades effect remains increasing, but the marginal trades effect is decreasing, since the distance between the buyers' valuation and the marginal seller's valuation is higher when c is lower.. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

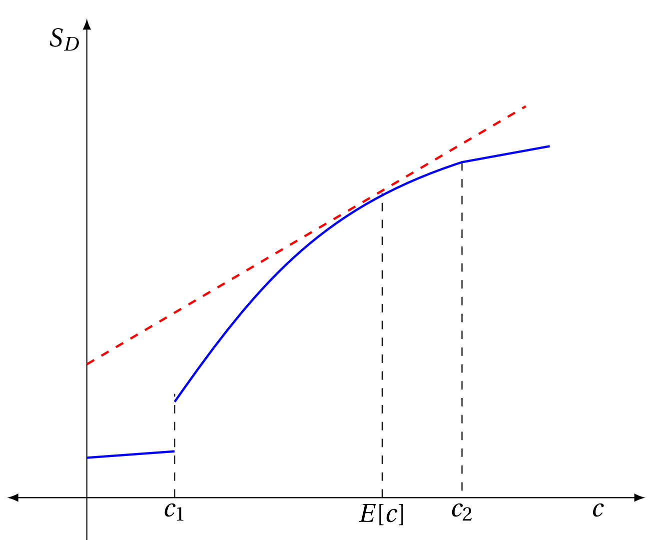

Fig. 2. The blue (solid) curve represents $S_D(c)$. The prior is such that, in the intermediated market, some sellers retain their good assets ($c_1 \leq \mathbb{E}c < c_2$). Condition ($\star$) ensures concavity in the region $c \geq c_1$. However, in this example, $\underline{c} < c_1$. The additional requirement ($\star\star$) guarantees that $S_D(c)$ lies everywhere below its tangent at $c = \mathbb{E}c$.

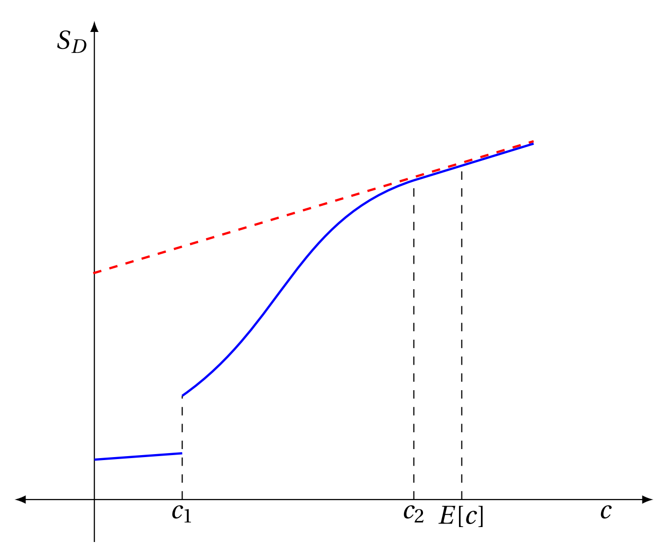

Fig. 3. In this case, the function $S_D(c)$ is not concave in $[c_1, c_2]$. The prior is such that, in the intermediated market, all sellers with good assets will sell ($\mathbb{E}c > c_2$). Moreover, the condition stated in part I of Theorem 1 holds. Thus $\mathbb{E}S_I(c) > \mathbb{E}S_D(c)$.

(I). (Uniform shifts). A uniform upward (downward) increase in sellers' values — i.e., $p(x)$ is constant — increases (decreases) the value of intermediation. An upward (downward) increase in buyers' values decreases (increases) the value of intermediation.

(II). (Lower inequality in sellers' valuations). An increase in sellers' values that is negatively and directly proportional to their type — i.e., $p(x) > 0$, $p'(x) < 0$, and $p''(x) = 0$) — increases the value of intermediation.

The main source of intuition behind the result is equation (7). When all sellers have higher private valuations, gains from trade are lower, worsening the adverse selection problem: the marginal trades effect is larger. The same is true when the buyers' valuation decreases. If sellers' valuations are more evenly distributed, the marginal seller becomes more sensitive to equal price changes: there is larger entry and exit in response to changes in c, amplifying the marginal trades effect and hence the value of intermediation.

4.2.2. Distribution of the shock

To end the comparative statics section, the next statement considers perturbations that induce mean-preserving spreads (or contractions) in the distribution $F(c)$.

Proposition 4 (*Changing risk in the shock*). *If condition* ($\star$) *holds and* $\underline{c} \geq c_1$, *more (less) risk in the shock* c *increases (decreases) the value of intermediation.*

Observe that, since intermediaries and sellers are risk-neutral and make decisions before c realizes, mean preserving changes in the distribution of c do not affect expected surplus in the intermediated market. By contrast, in the direct market, the shock c affects the final assets' allocation. When surplus is concave in c, adding risk in c reduces expected surplus. In relative terms, the intermediated market becomes more valuable.

4.3. Exclusivity in the intermediated market

Throughout the analysis, I have assumed that if sellers reject the intermediary's offer, they must retain the asset. In other words, intermediaries have *exclusive* access to buyers. Such a structure would emerge endogenously if exclusivity could be contracted upon ex-ante, before sellers obtain private information on their types.[18] While contracts mandating exclusive trading relationships are often used in practice, an alternative interpretation is to view such agreements as informal arrangements that are maintained throughout a long-term relationship during which sellers may wish to sell multiple assets repeatedly. In either case, the additional value that intermediaries bring to the market amounts to the difference $\mathbb{E}\left[S_I(c) - S_D(c)\right]$. The current model assumes a large supply of competitive intermediaries, so that sellers extract the entire surplus from the relationship. However, various forms of imperfect competition would give rise to a more even split. Therefore, the comparative statics results in Propositions 3 and 4 lend themselves naturally to empirical studies aimed at understanding the sources of intermediation rents across different markets (Section 7).

5. Relaxing exclusivity: The case of costly search

What would happen if sellers could bypass intermediaries and trade directly with buyers? The deviation would appeal to the marginal seller in the intermediated market, whose private valuation for the asset equals the intermediary's offer: for them, moving to $t = 1$ could not possibly lead to a payoff lower than what they receive in equilibrium.

I introduce a classic form of search frictions that reduces the sellers' incentives to search for buyers, thus supporting a market in which some trades occur via intermediaries.

The friction. Assume that to meet buyers, sellers must endure a search cost, denoted ψ. Intermediaries can search more effectively. For simplicity, they can meet buyers at no cost.[19]

Timing. At $t = 0$, sellers privately observe (x, θ), as well as the price offer made by competitive intermediaries. If they do not accept the offer, they choose whether to hold the asset until the end of period $t = 1$ or to search for buyers at $t = 1$. In the latter case, sellers, after paying ψ, observe the shock c as well as the competitive buyers' price offer.

Equilibrium prices and buyers' beliefs. As in the baseline model, intermediaries and buyers act competitively. I assume anonymity: buyers cannot tell sellers and intermediaries apart, but take into account that the measure of assets available for trade at $t = 1$ depends on c according to the strategy profile at $t = 0$.

Discussion and interpretation. In Appendix A.4, I relax anonymity and allow the choice of searching at $t = 1$ to convey a signal about the seller's type: in that framework, a fully intermediated market can be supported even when $\psi = 0$. Moreover, when $\psi > 0$ equilibria exist in which some sellers separate by searching for buyers. The potential for types' separation represents an *additional* benefit of the presence of intermediaries that I mute down in the model below. I do so for three reasons: first, in many markets buyers cannot distinguish whether their counterparty is delegated to execute a trade or trading on its own behalf;[20] second, by muting down the signaling effect, the analysis isolates the novel mechanism of the paper;[21] third, the setup fits best the applications discussed in Section 8. Throughout the analysis, I use a welfare criterion that incorporates the search costs incurred by sellers. That is:

welfare = surplus - search costs.

Formally, denote with $W(\psi)$ aggregate welfare as a function of the parameter ψ.

To simplify the exposition, I assume in this section a stronger form of regularity ensuring that the valuation of the marginal seller at $t = 1$ is continuous in c.

Assumption 4 (*Strong regularity*). For all x, $u'(x) > \vartheta'(x)$.

Extreme search costs. A partial characterization of the equilibria that emerge depending on the cost ψ helps interpret the analysis of Sections 3 and 4.

Proposition 5 (*Extreme search costs and welfare comparison*).

(I). If $\psi = 0$, there is a continuum of (welfare equivalent) equilibria in which only sellers with valuation below $u\left(x^*\left(\underline{c}\right)\right)$ sell to the intermediaries with arbitrary probability.

(II). There exist two thresholds for ψ, denoted $\underline{\psi}$ and $\overline{\psi}$ such that, for $\psi > 0$

- *If $\psi \leq \underline{\psi}$, only sellers with valuation below $u\left(x^*\left(\underline{c}\right)\right)$ sell to the intermediaries.*
- *If $\psi \geq \overline{\psi}$, all trades are intermediated.*

[18] In a similar spirit, Glode and Opp (2016) outline a "network-formation and trading" game in which agents can enter deal-flow agreements specifying for each agent a unique direct counterparty from which to buy the asset, as well as transfers to be made before trades take place. A formal analysis is omitted for brevity.

[19] A preference for early trades would give the same qualitative results. In some models, sellers suffer "holding shocks" which they can avoid by selling early; typically, intermediaries are assumed to be less exposed to such shocks, thus able to offer "immediacy". One could interpret ψ as the extra disutility from holding the asset. In this case, the cost ψ would also be borne by sellers who do not search for buyers, and decide to retain the asset until the end of $t = 1$. Stronger frictions would imply larger delays, hence could be captured by an increase in ψ.

[20] Even if they did, equilibria in which buyers apply a premium or a discount to intermediaries are vulnerable to various forms of joint deviations by pairs of sellers and intermediaries.

[21] A vast literature already exists that explores the scope for signaling quality in markets with adverse selection (see Leland and Pyle 1977, Janssen and Roy 2002 and the recent Fuchs et al. 2022). When they can assess quality, intermediaries can act as certifiers, reducing information frictions (see Biglaiser 1993 and the recent Alti and Cohn 2022).

(III). *For $\psi \leq \underline{\psi}$, total expected surplus is the same as in a setup in which all trades are direct. Hence, under the conditions identified in Theorem 1,*

$$\mathbb{E}[W(\psi)]_{\psi \geq \overline{\psi}} > \mathbb{E}[W(\psi)]_{\psi \leq \underline{\psi}}.$$

A sufficiently large ψ deters all sellers from searching for buyers. Sellers are left with the choice whether to accept the intermediary's offer or hold the asset until the end of $t = 1$. At the other extreme, a sufficiently small ψ induces all sellers with a good asset to bypass the intermediary and observe the buyers' offer. The resulting allocations replicate, respectively, the intermediated and direct market introduced in Section 2. Under the conditions identified in Theorem 1, sellers are worse off in the second scenario.

The main implication is that eliminating search frictions altogether worsens allocative efficiency and reduces total welfare.

Moderate search costs. For a fixed ψ, can the direct beneficial effect of marginally reducing search costs be dominated by the indirect negative effect of discouraging intermediation? Perhaps surprisingly, the answer is positive under condition ($\star$). To establish this result, I first describe the equilibrium when $\psi \in [\underline{\psi}, \overline{\psi}]$. A seller who anticipates accepting every offer at $t = 1$ has no incentive to endure the search cost, and strictly prefers to sell to the intermediary at $t = 0$. Likewise, a seller who anticipates rejecting every offer at $t = 1$ also has no incentive to endure the search cost, and strictly prefers to retain the asset. Reaching out to buyers entails an option to act upon c, which is most valuable for sellers with intermediate private valuation.

Proposition 6 (*Equilibrium when search costs are moderate*). *If $\psi \in [\underline{\psi}, \overline{\psi}]$, all sellers with bad assets sell to intermediaries at $t = 0$. The strategies of sellers with good assets form three intervals:*

- *if x lies in $[0, X_0)$, the seller accepts the intermediaries' offer at $t = 0$;*
- *if x lies in $[X_0, X_1)$, the seller endures the search cost and sells if $p^b(c) \geq u(x)$;*
- *if x lies in $[X_1, 1]$, the seller holds the asset until the end of $t = 1$.*

The thresholds X_0 and X_1 are unique, continuous functions of ψ. The function $X_0(\psi)$ is strictly increasing, $X_1(\psi)$ is strictly decreasing, and $X_0(\overline{\psi}) = X_1(\overline{\psi}) = x^*(\mathbb{E}c)$.

A marginal decrease in ψ leads to *disintermediation*, a decrease in the measure of sellers who accept the intermediaries' offer (X_0 decreases). Moreover, more sellers choose to search rather than keeping their asset (X_1 increases). The supply of good assets at $t = 1$, which is bounded below by λX_0 and above by λX_1, exhibits more pronounced fluctuations: as search frictions reduce, states in which the demand is low are associated to even less trade, and a lower price, reflecting lower average asset quality; states in which the demand is high are instead associated to even more trade and a higher price. It turns out that condition ($\star$) determines the net effect on average traded asset quality, allocative efficiency and welfare, leading to the result below.

Theorem 2 (*Assets quality, total welfare and search costs*). *If ($\star$) holds for all $x \in \chi$,*

(I). *Average traded assets' quality is weakly increasing in ψ, and strictly so if $\psi \in [\underline{\psi}, \overline{\psi}]$;*

(II). *Expected welfare $\mathbb{E}W(\psi)$ is continuous and non-monotone in ψ. If $\psi \in [0, \underline{\psi})$ it is strictly decreasing; if $\psi \in [\underline{\psi}, \overline{\psi}]$, it is strictly increasing and for all $\psi \geq \overline{\psi}$ it is constant. Finally, $\mathbb{E}W(\psi)$ is maximized at all values $\psi \geq \overline{\psi}$ (see Fig. 4).*

To build intuition for part (I), note that the buyers' price reacts more strongly to an equal increase in the supply of good assets in low states, where average assets' quality is lower.

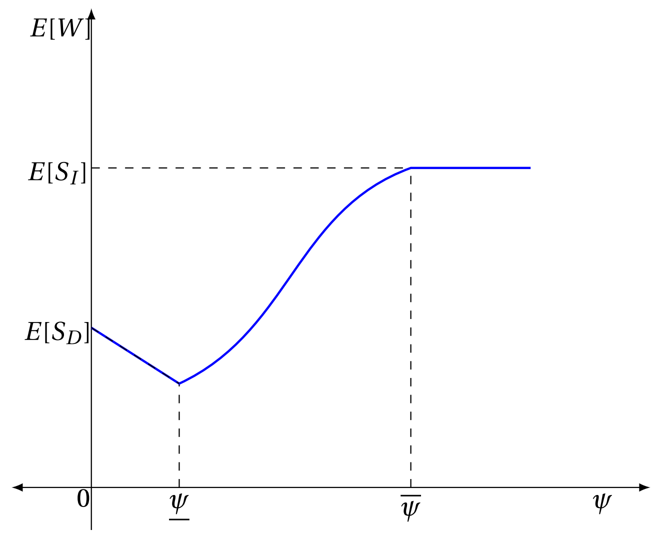

Fig. 4. The picture illustrates the effect of ψ on total welfare when ($\star$) holds. At $\psi = 0$, all trades are non-intermediated but search costs are zero, therefore $\mathbb{E}W = \mathbb{E}S_D$. For sufficiently large ψ, all trades are intermediated, $\mathbb{E}W = \mathbb{E}S_I$. Since ($\star$) holds, welfare is higher in the second case (Proposition 5). If $\psi \leq \underline{\psi}$, trades are non-intermediated but sellers bear positive search costs, hence welfare decreases in ψ. If $\psi \in [\underline{\psi}, \overline{\psi}]$, a decrease in ψ leads to disintermediation, which decreases welfare according to Theorem 2.

To understand part (II), decompose the effect of search costs on welfare as:

$$\frac{d\mathbb{E}W(\psi)}{d\psi} = \lambda \underbrace{\frac{dX_0}{d\psi}\frac{d\mathbb{E}W}{dX_0} + \frac{dX_1}{d\psi}\frac{d\mathbb{E}W}{dX_1}}_{\text{marginal effect}} - \underbrace{\lambda\left(X_1 - X_0\right)}_{\text{inframarginal effect}}$$

and observe that, if assets were traded at fair price, sellers' decisions would be socially efficient, hence $\frac{d\mathbb{E}W}{dX_0} = \frac{d\mathbb{E}W}{dX_1} = 0$. The only effect of the increase in ψ would be the direct increase in search costs borne by all inframarginal sellers who continue to search. By contrast, when asymmetric information causes assets to trade at a discount, the seller who is indifferent between selling to the intermediary and searching does not internalize the full surplus generated by trading with final buyers at $t = 1$. Indeed, X_0 solves the indifference condition:

$$\psi = \underbrace{\int_{c \leq c(X_0)} u\left(X_0\right) - \left[\vartheta\left(X_0\right) + c\right] dF}_{\text{private value of search}} \tag{8}$$

where $c\left(X_0\right)$ denotes the state in which, ex-post, type X_0 is indifferent whether or not to accept the buyer's offer. The welfare effect of a marginal change in X_0 is then:

$$\frac{d\mathbb{E}W}{dX_0} = \int_{c \leq c(X_0)} v - \vartheta\left(X_0\right) dF.$$

A similar logic can be used to derive the welfare effect of a marginal change in X_1. The positive effect of an increase in X_0 is larger than the negative effect of an equal decrease in X_1, because at the margin, gains from trade are larger in low states, when the marginal seller has a low valuation. Under ($\star$), this effect is so pronounced to dominate the inframarginal effect of an increase in search frictions.

The role of intermediaries. To qualify the effect that intermediaries have in the model, I characterize equilibrium and welfare under a scenario in which intermediaries are not available—sellers cannot trade at $t = 0$.

The seller's strategy boils down to the decision whether to search for a buyer or retain the asset until the end of $t = 1$. Retaining is more valuable for sellers with a higher private valuation, leading to the characterization below.

Proposition 7 (*Equilibrium without intermediaries*). *There exists a threshold, $\widehat{\psi}$, such that, in the market without intermediaries:*

- *For $\psi \leq \widehat{\psi}$, sellers with bad assets and those with good assets and type $x \leq X_1(\psi)$ search for buyers, while all others retain the asset.*
- *For $\psi \in \left[\widehat{\psi}, \mathbb{E}c\right]$, only sellers with bad assets search for buyers.*
- *For $\psi > \mathbb{E}c$, all sellers retain their assets.*

Therefore, average traded assets' quality is (weakly) decreasing in ψ.

The main message is that the benefits of search frictions identified in Theorem 2 only obtain in markets in which intermediaries are available.

Introducing intermediaries has the following effects. First, when $\psi \leq \overline{\psi}$, some sellers who would search in the absence of intermediaries, sell at $t = 0$ when intermediaries are present, thereby saving on search costs. This has a direct positive effect on welfare, but also implies a different allocation of the asset ex-post: while intermediaries sell the asset in all states, some sellers who search choose to retain if demand is sufficiently low. Second, recall that in the region $\psi > \overline{\psi}$ no seller searches in the intermediated market. Among those who sell to the intermediary, some would choose to retain in case intermediaries were not present. The next result compares welfare under the two scenarios. I denote $W_N(\psi)$ welfare in case there are no intermediaries, for a given ψ.

Proposition 8 (*The value of intermediaries*). *For all $\psi > 0$:*

(I). When intermediaries are present, the volume of assets sold to final buyers is higher in every state c.
(II). Intermediaries strictly improve welfare: $\mathbb{E}W(\psi) > \mathbb{E}W_N(\psi)$.
(III). In some cases, the difference $\mathbb{E}\left[W(\psi) - W_N(\psi)\right]$ exceeds the additional search costs borne by sellers in the market without intermediaries.
This is always true if $v + c > u(x)$ for all c and x.

Part *(I)* is a consequence of the fact that the assets acquired by intermediaries at $t = 0$ are resold in every state. To understand part *(II)*, observe that in the market with intermediaries the cost of search equals the marginal seller's private option value of observing the buyers' offer. Thus, even when the seller's choice to sell to the intermediary entails some inefficient trade ex-post, such a misallocation *cannot* exceed the benefits of reduced costly search. Finally, part *(III)* is best understood in the context of the special case of universal gains from trade ($v + c > u(x)$ for all c and x). In such a case, the presence of intermediaries, by inducing more trade, improves surplus and hence increases welfare beyond the savings in search costs. Clearly, while sufficient, the condition is not necessary for the result to hold.

6. Delegation contracts

The intermediated market permits sellers to commit the decision whether or not to trade before observing the state of demand. The same would also be possible if intermediaries received from sellers a mandate to accept the buyers' offer. More generally, consider contracts which specify the states in which intermediaries are committed to accept the buyers' offer, and those in which sellers have the option to reject it. Below I provide conditions under which, within this class, the surplus-maximizing contract restores sellers' discretion in low demand states.

A *contract* is a function σ, mapping the realization of the shock into two outcomes, labeled $\{0, 1\}$. Formally, $\sigma : \left[\underline{c}, \overline{c}\right] \to \{0, 1\}$. The case $\sigma(c) = 1$ indicates that the intermediary is committed to sell. The case $\sigma(c) = 0$ indicates that the seller can observe the buyer's offer and decide whether or not to accept it. When the asset is sold, the seller receives the proceeds. Following the structure of the intermediated market described in Section 2, I assume that at $t = 0$ sellers can observe their type and decide whether to enter the contract with the intermediary or retain the asset until the end of $t = 1$.

Remark 4 (*Special cases*). The case $\sigma(c) = 1$ for all c replicates the outcome of the intermediated market. The case $\sigma(c) = 0$ for all c replicates the outcome of the direct market.

The result below characterizes the contract that maximizes total expected surplus. Because of competition among buyers, it also maximizes the seller's expected payoff.[22]

Proposition 9 (*Partial delegation contracts*). *Assume that $(\star)$ holds. Then, the surplus-maximizing contract takes a partitional form: sellers have discretion ($\sigma(c) = 0$) if and only if $c \leq c'$ for a threshold $c' \in \left[\underline{c}, c_1\right)$. Moreover,*

(i). In states $c \leq c'$, sellers accept the buyers' offer if and only if $\theta = b$.
(ii). If $\underline{c} \geq c_1$ or $(\star\star)$ holds, $c' = \underline{c}$ (full delegation). Otherwise, $c_1 > c' > \underline{c}$ (partial delegation).
(iii). Sellers enter the contract if either $\theta = b$ or $\theta = g$ and $x \leq x^*(\hat{c})$ with $\hat{c} := \mathbb{E}\left[c | c \geq c'\right]$.
All other sellers retain the asset.

The characterization of the threshold c' is in the proof. The key insights of the analysis are best presented in the form of comparisons with the baseline scenario of the intermediated market introduced in Section 2, which corresponds to the special case $c' = \underline{c}$. These comparisons lead to Corollaries 1, 2 and 3.

Observe from *(iii)* that the marginal seller participating in the contract is the marginal seller trading in the direct market if the realized shock equals $\hat{c} := \mathbb{E}\left[c | c \geq c'\right]$. The case $c' = \underline{c}$ implies that $\sigma(c) = 1$ for all c and hence by Remark 4 it replicates the intermediated market. In some cases (see part *(ii)*), it emerges as the surplus-maximizing contract. Since $c' > \underline{c}$ implies that $\hat{c} > \mathbb{E}c$, and since $x^*(\hat{c})$ is increasing, more sellers accept the partial delegation contract compared to the baseline, full delegation contract. Higher participation comes at the cost of fewer trades in low-demand states.

Corollary 1 (*Participation and ex-post trade*). *Compared to the baseline (full delegation) case, more good asset holders enter the contract. Therefore, more good assets are traded in states $c \geq c'$. However, fewer good assets are traded in states $c < c'$. Thus, the price that buyers pay in states in which $c \geq c'$ is higher, whereas it is lower in states $c < c'$.*

How does the contract affect sellers' expected payoff, given their assets quality and private values? The partial delegation contract gives high-valuation sellers downside protection, inducing higher participation. In turn, higher participation alleviates adverse selection and thus increases the buyers' offer in all states in which intermediaries are committed to accept it. While this feature increases overall surplus and hence the sellers' expected payoff in aggregate, note that it is those sellers with a larger option value of rejecting the buyers' offer who benefit the most from it. By contrast, sellers with sufficiently low valuation benefit from the higher price when $c \geq c'$, but may be hurt from the lower price when $c < c'$.

[22] To simplify the exposition, I do not explicitly model the negotiation between sellers and intermediaries. It can be shown that the same contractual arrangement would emerge as the equilibrium of a game in which intermediaries compete by posting contracts that sellers may enter before observing their types. In equilibrium, the contracts would entail no transfer to the intermediary.

Corollary 2 (*Winners and losers*)**.** *Assume that the surplus-maximizing contract entails partial delegation. Denote with $\Delta(x,\theta)$ the difference in (x,θ)-seller's expected payoff under the surplus-maximizing contract compared to the baseline full delegation contract. It holds that: $\Delta(.,b) \leq \Delta(0,g)$ and that $\Delta(x,g)$ is continuous and single-peaked at $x = x^*(\mathbb{E}c)$, with $\Delta(x^*(\mathbb{E}c),g) > 0$. In some cases, $\Delta(x,g) < 0$ for sufficiently small x. Finally, $\Delta(x,g) = 0$ for all $x > x^*(\hat{c})$.*

Intuitively, when the optimal contract is available, sellers with good assets who would also accept the baseline contract — those with $x \leq x^*(\mathbb{E}c)$ — retain their assets more often, obtaining $u(x)$ in all states in which $c < c'$. By contrast, sellers who accept the optimal contract but would reject the baseline — those with $x \in [x^*(\mathbb{E}c), x^*(\hat{c})]$ — retain their asset in a smaller set of states. This explains why the benefit of partial delegation is non-monotone in the sellers' private valuation, and could even be negative for sellers with sufficiently low private valuations.

Does the optimal contract implement a more efficient allocation in all states? A basic optimality argument suggests this is not the case. Ultimately, the partitional form of the surplus-maximizing contract emerges because in states below c_1, due to market unraveling, letting sellers react to the price offer has a limited downside, but has the benefit of attracting more good asset holders in the contract. At the optimal threshold c', the marginal cost of letting sellers reject must be positive.

Corollary 3 (*Inefficient rationing*)**.** *When partial delegation is optimal ($c' > \underline{c}$), some good assets are inefficiently retained in states $c \leq c'$. That is, for some $c \leq c'$ and some $x' \leq x^*(\hat{c})$, it holds that $v + c > u(x)$ for all $x \leq x'$. In the baseline (full delegation) case, these assets would have been efficiently allocated to final buyers in all such states.*

If intermediaries were committed to accept every offer at $t = 1$, low-valuation sellers will never retain the asset. Thus the surplus-maximizing contract further distorts the asset's allocation in low-demand states, compared to what happens in the intermediated market presented in Section 2.

The shape of the optimal contract echoes general insights from the literature on delegation (Holmstrom, 1980) and is also reminiscent of the optimal "buyback" contracts emerging in setups where intermediaries privately observe demand, and have incentives to misreport it (Arya and Mittendorf, 2004). Notably, my model does not feature any agency problem between the seller and the intermediary.

Further directions. The contract implements a solution whereby sellers behave *as if* they only observed a binary signal of the shock (in the spirit of persuasion la Kamenica and Gentzkow (2011)), before choosing whether or not to accept the best buyers' offer. The analysis naturally generalizes the baseline model and isolates the novel role of intermediation that my paper brings forward: intermediaries mediate information over the gains from trade. Moving beyond such a role, a larger contract space would likely lead to further improvements. The function σ is assumed not to depend on the sellers' type (x,θ), which accommodates the case in which the intermediary cannot observe (x,θ). However, relaxing the restriction would clearly present benefits. Indeed, while not necessarily optimal, making sellers commit to trade *before they obtain any private information* would overcome the lemons problem. Moreover, a contract may specify transfers that depend on c. To avoid the inefficiency described in Corollary 3 and screen sellers with different private valuations, a transfer from the intermediary to the seller in states $c \leq c'$ could prevent low-valuation sellers from pulling out of trade, keeping the benefits of downside protection to high-valuation sellers. These extensions would introduce new trade-offs, and a characterization is beyond the scope of this paper.[23]

7. Empirical predictions

I summarize the empirical predictions that the model generates, emphasizing those that are unique to my model and in contrast with existing models of intermediation.

Predictions 1 and 2 are presented as comparisons between an intermediated and a direct market. However, they could also be interpreted as comparisons between markets that vary in the proportion of trades conducted through intermediaries, say because of differential search frictions or intermediaries' inventory costs (see Sections 5 and 8.2).

Prediction 1. *Average quality and volume of traded assets and the buyers' price are less sensitive to shocks to gains from trade in the intermediated market, relative to the direct market.*

Note that Prediction 1 emerges from the model regardless of whether the conditions of Theorem 1 hold. The effect it highlights can be viewed as a novel form of liquidity provision by intermediaries: intermediaries mitigate the effects of public shocks to gains from trade on prices and, therefore, smooth out the part of fluctuations in trade volume that are caused by the interaction of these shock with the severity of adverse selection.

Prediction 2. *Unconditionally on shocks to gains from trade that occur before assets are sold to buyers, assets traded in the intermediated market have higher quality and are sold at a higher price, relative to the direct market. When gains from trade reveal to be high, the opposite holds.*

The fact that *on average* intermediaries sell at a higher price emerges in setups in which intermediaries provide immediacy (Rubinstein and Wolinsky, 1987; Duffie et al., 2005), are endowed with superior bargaining skills (Farboodi et al., 2025) or serve a quality-certification role (Biglaiser, 1993). If intermediaries cream-skim (Bolton et al., 2016), they would also sell at a higher price, and operate in markets in which average asset's quality is higher. However, none of the setups above predict differential comparisons conditional on shocks to traders' private values such as stated in Prediction 2. To the best of my knowledge, this prediction has not been tested empirically.

Predictions 3 and 4 derive from a model in which sellers can establish relationships with intermediaries, and intermediaries can extract some of the surplus from the relationship, for example in the form of fees (see Section 4.3).

Prediction 3. *In markets in which sellers' (buyers) valuations are higher (lower), intermediaries earn higher rents.*

Prediction 3 pertains to differences in the predictable component of sellers' and buyers' valuation for the asset. Note that models of intermediation based on search frictions (Duffie et al., 2005) would predict the opposite: when gains from trade are low, the relative value of speeding up trade is lower, reducing traders' willingness to pay for the intermediaries' services.

Prediction 4. *In markets in which shocks to gains from trade are ex-ante more volatile, intermediaries earn higher rents.*

In classic inventory models intermediaries obtain larger markups when buyers' demand is more risky, and this is proportional to their

[23] With a flexible transfer schedule and assuming that intermediaries must break-even in expectation, setting a transfer from the seller to the intermediary in all states $c \leq c'$ (causing all good asset owners to withdraw from trade) has an advantage: the collected transfers allow intermediaries to make a positive transfer to sellers in higher state, inducing more trade in high-demand states.

inventory costs (Stoll, 1978). By contrast, in my model volatility in demand increases sellers' utility from intermediation, thereby increasing the rents that intermediaries can extract irrespective of their risk-bearing capacity.

8. Applications

Although deliberately stylized, the model offers insights on the role of intermediaries in markets with adverse selection. Below, I discuss two examples.

8.1. The leveraged loans market

Leveraged loans are below-investment-grade syndicated loans issued to firms (in my setup, the *sellers*) and originated and underwritten by a lead arranger, often a bank (the *intermediary*) which subsequently sells them to institutional investors (the *buyers*), typically collateralized loan obligations (CLOs). The increased participation of such institutional investors in the market for corporate credit, along with banks' growing tendency to offload larger shares of such risky loans instead of retaining them on their balance sheets, raises several questions. What is the economic role of lead arrangers adopting the "underwrite-to-distribute" model? And what would be the implications, should regulatory pressure cause banks to abandon underwriting and act solely as brokers? My model offers a novel perspective on these questions.

Mapping the model to the application. In my model, intermediaries buy the asset at $t = 0$, which corresponds to banks underwriting the loans they arrange. The shocks to buyers' valuations can capture shifts in investors' demand—driven either by capital flows to such funds (Ivashina and Sun, 2011) or CLO's industry-level portfolio constraints that affect loan demand independently on borrower quality. While it is crucial to assume that, realistically, borrowers are more informed than final buyers about their own quality and about their funding needs, the mechanism behind my model works regardless of whether intermediaries are informed.[24] Contractually, borrowers often remain in part exposed to price risk through automatic "flexes" in the interest rate in response to low demand. Presumably, this feature mitigates banks' risk exposure. However, in the model that I present, the way in which sellers and intermediaries share price risk is irrelevant—all results would hold if intermediaries offered a price that is a function of c, as long as their payment equals the resale price in expectation. Finally, arrangers may retain the loan when demand is particularly low, a form of rationing absent in my model. This element can help elicit demand truthfully in the book-building process (Bruche et al., 2020). A simplifying feature of my model is that competing buyers have homogeneous preferences and bid their true valuation.

Novel insights. The key observation is that investors anticipate the banks' motive to sell underwritten loans despite a low price—that is to avoid keeping such loans in their balance sheet, eroding regulatory capital. As firms contract with banks before demand realizes, the pool of borrowers is fixed and does not depend on the ultimate price terms that the banks will agree with final investors. When shocks depress the demand and increase rates, investors do not adjust downwards their beliefs about a borrowing firm's quality. Essentially, the arrangement breaks the classic adverse selection mechanism whereby higher rates attract lower-quality borrowers. The theory has important implications. Recent evidence that banks often sell the entire stake of the loans they originate (Blickle et al., 2020) appear inconsistent with theories in which banks signal quality via retention (Leland and Pyle, 1977), or improve quality via monitoring. But in light of the effects that I uncover, this evidence need not imply that information frictions are not present, and that banks do not play an important role in mitigating them.

Using the discussion in Sections 4 and 5, an arrangement in which banks underwrite-to-distribute may emerge if firms can commit ex-ante to such a relationship or because firms value funding certainty at $t = 0$ (a motive that would be captured by a large ψ in the model variant with search frictions). In both cases, the model highlights the risks of a shift toward a brokered market where banks no longer underwrite or where borrowers deal more directly with investors, as seen in the growth of private debt.[25] Recently, banks have been forced to sell underwritten loans at discounted prices as macroeconomic shocks sharply reduced investors' demand.[26] According to my theory, in a counterfactual market without intermediaries, similar shocks would have resulted in a sharper contraction in credit supply, driven not just by pricing, but by amplified adverse selection.

8.2. Dealers in OTC markets: Matchmaking vs market-making

Since the 2008 financial crisis, many OTC markets have featured a dual form of intermediation. Dealers provide immediacy when they act as market-makers, buying the asset and holding it in their inventory in what is referred to as a "principal trade". Alternatively, they match sellers and buyers in an "agency trade". In the latter case, sellers bear delay costs and are left exposed to price variations that may lead them to choose not to trade if the demand turns out to be low (Goldstein and Hotchkiss, 2020). The corporate bonds market is a primary example.

Mapping the model to the application. Secondary trades of corporate bonds are infrequent, often characterized by asymmetric information (Ronen and Zhou, 2013; Jiang and Sun, 2015; Wei and Zhou, 2016), and dominated by dealers performing the dual function described above. This makes it natural to model the market as the one with search frictions I describe in Section 5, up to some minor modifications. Assume that for reasons outside the model, trade must occur through dealers (as often assumed by the literature, e.g. Sambalaibat 2022). Sellers at $t = 0$ observe their type and choose between a principal trade — the dealer buys the asset outright — and an agency trade — the seller must wait until $t = 1$, when the dealer finds a buyer. In the latter case, after observing the buyer's offer, the seller chooses whether to accept it or retain the asset. The variable ψ introduced in Section 5 can capture the cost of holding the asset that is borne by all asset owners unless they sell to dealers in principal trades. Assume that, for principal trades, dealers bear an inventory cost ι. In line with Section 5, buyers cannot distinguish whether the dealer is selling from its inventory or simply matching a seller's order. However, buyers correctly anticipate the differential sorting of sellers in the two trading modes. Dealers' aversion to holding inventory is a source of transaction costs, measured as the difference between the price at which assets can be bought from and sold to dealers. In a principal trade, a competitive dealer bids at $t = 0$:

price of principal trade = expected resale price - inventory cost.

Thus, for principal trades, transaction costs equal the inventory cost ι on average. For agency trades, competitive bidding implies that transaction costs are zero.[27]

[24] The case in which intermediaries and final buyers are symmetrically informed appears realistic in light of recent evidence. Indeed, Bruche et al. (2023) find that banks learn about borrower quality from investors in the book-running and negotiation process. The practice that arrangers share their information with investors in the book-building phase (described for instance in Hinzen 2023) supports the interpretation that asymmetric information between arrangers and investors is likely small.

[25] A growing market so far largely confined to smaller and riskier firms (Block et al., 2024)

[26] See https://www.ft.com/content/ceff8fd1-a3ef-4bce-9955-59a74bc0d226 .

Novel insights. Below, I summarize the main insights that derive from this application. Detailed derivations are in Appendix A.

Remark 5. The setup can be analyzed using the characterization in Section 5, which corresponds to the special case: $\iota = 0$, with the difference that sellers do not have the option to retain the asset without suffering the cost ψ. For a range of parameters and $\iota > 0$:

(I). Only bad asset owners and good asset owners with low valuation choose principal trades.
(II). An increase in dealers' inventory costs:
  (i). decreases the volume of principal trades and increases the volume of agency trades;
  (ii). causes a more pronounced reduction in trade volume when demand is low;
  (iii). decreases average traded assets' quality, the expected volume of trade, allocative efficiency and welfare.
(III). The volume of principal trades decreases continuously with inventory costs, and is zero when they are sufficiently high.

Sellers with bad assets and those with higher liquidity needs choose principal trades. An increase in inventory costs causes a shift from principal to agency trades that reduces the expected volume of trade, even without accounting for delay: this reinforces the argument, advanced by Kargar et al. (2021), for distinguishing between the two forms of intermediation to better assess market liquidity and its deterioration during crises. The increase in inventory costs has a direct positive impact on transaction costs, but because it leads some sellers to switch to agency trades, the overall effect on average transaction costs is ambiguous, and may be negative, consistent with the mechanism in Saar et al. (2023). Unlike in their model, higher inventory costs always reduce welfare because of heightened adverse selection: sellers opting for agency trades inefficiently refrain from trading in low-demand states. In offering a novel framework in which the severity of adverse selection in the corporate bond market depends on dealers' inventory capacity, the model can inform the literature that investigates the effects of regulations on dealers' liquidity provision (Bao et al., 2018).

9. Conclusions

This paper highlighted a novel source of efficiency gains stemming from the presence of intermediaries in markets plagued by adverse selection. By trading first with an intermediary, the owner of an asset with high fundamentals commits not to keep it in states in which gains from trade reveal, ex-post, to be low. These states would otherwise be associated to severe adverse selection, preventing also sellers with low idiosyncratic valuations from trading.

I used the theory to revisit the role of lead banks in the market for leveraged loans and to generate insights on the effects of search frictions and dealers' inventory capacity in the functioning of many decentralized assets markets, such as that for corporate bonds.

CRediT authorship contribution statement

Francesco Sannino: Writing – review & editing, Writing – original draft, Methodology, Investigation, Formal analysis, Conceptualization.

[27] In my model, dealers are indifferent in equilibrium between offering the two forms of intermediation, unlike in An and Zheng (2023), who model dealers' incentive to prioritize inventory turnover.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Appendix A. Extensions and alternative specifications

A.1. Intermediaries private value

When u^i is low. If intermediaries anticipate to accept the best buyer's offer in every state, they behave at $t = 0$ just as if they have no value for holding the asset. Bidding up to the asset's resale price, their offer is accepted by all sellers with the bad asset and all sellers with a good asset and private valuation below $u(x^*(\mathbb{E}c))$. In equilibrium, such a belief would be consistent if the price at $t = 1$ was higher than the intermediaries' valuation u^i when the shock is lowest, that is at $c = \underline{c}$.

Proposition 10 (*Intermediaries with low valuation*). *If $u^i \leq \frac{x^*(\mathbb{E}c)\lambda}{x^*(\mathbb{E}c)\lambda+1-\lambda}\upsilon + \underline{c}$, the intermediated market's equilibrium allocation is equivalent to the case where $u^i = 0$. Hence, all results in Sections 3–6 continue to apply.*

The upper-bound on u^i depends on the endogenous $x^*(\mathbb{E}c)$. To have a condition on the model's primitives, one can replace it with its lower bound, $\underline{x}(\mathbb{E}c)$. Observe that, in the intermediated market, the supply of good assets at $t = 1$ is fixed at $x^*(\mathbb{E}c)$ for any realization of c, so long as u^i is sufficiently small. The higher the expected shock is, the higher is the upper-bound on u^i necessary to support the equilibrium where intermediaries always sell at $t = 1$.

To end the analysis of this case, the example in the remark below clarifies the sense in which the restriction on u^i is not strong.

Remark 6. Take Example 2 (where u are distributed uniformly in $[2, 4]$) and set $\underline{c} = 2.1$ and $\mathbb{E}c = 2.3$. Then the restriction in Proposition 10 reads: $u^i \leq \frac{x^*(2.3)}{x^*(2.3)+1}3 + 2.1 \simeq 3.46$, therefore intermediaries' valuation can be above the median seller's valuation.

The example highlights how the fact that intermediaries and sellers trade before c realizes is central to the paper's main result: since a large measure of sellers accepts the intermediaries' offer at $t = 0$, the offer that intermediaries receive at $t = 1$ is sufficiently large that the market does not unravel even when the shock is lowest, and despite intermediaries attach a relatively large private value to holding the good asset.

When u^i is high. Consider the intermediary's decision at $t = 1$. When it holds an asset of bad quality, the intermediary accepts every non-negative price offer. When it holds an asset of good quality, the intermediary accepts if and only if $p \geq u^i$. In the analysis that follows, such a condition need not hold in equilibrium. In particular, note that taking as given the sellers' strategy at $t = 0$, which is described by a cutoff, denoted X, buyers break-even condition implies that the best price offer an intermediary receives at $t = 1$, given a realization of c, is

$$p^b = \begin{cases} \frac{X\lambda}{X\lambda+1-\lambda}\upsilon + c & \text{if} \quad p^b \geq u^i \\ c & \text{otherwise} \end{cases}. \tag{A.1}$$

For every given X, equation $(A.1)$ identifies a cutoff c^i below which intermediaries with good assets choose not to sell them. At $t = 0$, intermediaries bid up to their valuation which is:

$$p^i = \mathbb{E}\max\left\{p^b, u^i\right\}$$

and hence X is a function of u^i as implicitly described by the indifference condition

$$u\left(X\left(u^i\right)\right) = \mathbb{E}\max\left\{\vartheta\left(X\left(u^i\right)\right) + c, u^i\right\}.$$

The following result states that, under conditions that must hold together with those stated in Theorem 1, the intermediated market generates higher expected surplus than the non-intermediated market for any value of u^i.

Proposition 11 (*Intermediaries with high valuation*)**.** *Assume that the conditions in Theorem 1 hold and that $u'(x) > \vartheta'(x)$. Then, if $\frac{F(c)(1-F(c))}{f(c)} > u'(x) - \vartheta'(x)$ for all c and x, and $\frac{F(c)}{f(c)} > v - \vartheta(1)$ for all c, the intermediated market dominates (i.e., $\mathbb{E}S_I(c) \geq \mathbb{E}S_D(c)$) for any value of u^i.*

Observe that the sufficient conditions on F in Proposition 11 are restrictive but not necessary to obtain that the intermediated market dominates. To construct cases where they hold, one can assume that the distribution of the shock has "fat" left and right tails.[28]

A.2. Intermediaries information

A.2.1. Information about asset's quality

When intermediaries' valuation for either asset is zero, as in the baseline specification, it is inconsequential whether they can tell high-quality assets from low-quality ones. Anticipating they will want to sell either asset at any non-negative price at $t = 1$, their own price offer at $t = 0$ will equal the asset's expected resale price, which does not depend on θ.

The same reasoning would apply when their valuation for the good asset is positive but sufficiently low. Conjecturing the marginal high-quality seller at $t = 0$ is some type $\tilde{x}$, those intermediaries who cannot observe the asset's quality will sell the asset they bought at a price p if and only if $p \geq \frac{\tilde{x}\lambda}{\tilde{x}\lambda+1-\lambda}u^i$. It is easy to see that this condition is even less stringent than the one identified in Proposition 10.

Remark 7. If the intermediaries' valuation is low as specified in Proposition 10, all results of the paper hold irrespective of intermediaries' information over the asset's quality.

The case where u^i is high and intermediaries could not observe θ would be more intricate due to the assumption, otherwise made for simplicity, that there are always gains from allocating the bad assets to final buyers. The specific configuration might cause uninformed intermediaries to inefficiently retain the bad asset in equilibrium.

To sum up, the model's key forces and the analysis remain largely unaltered when intermediaries are assumed not to observe the asset's quality. In some situations, the assumption would in fact strengthen the case for intermediation.

A.2.2. Information about the shock

What if intermediaries had access to superior information about the future realization of the shock? When the shock only affects the sellers' valuation (as in Appendix A.3), in the equilibrium of the intermediated market that I characterize, the buyers' price offer at $t = 1$ would not depend on its realization. Hence, uninformed intermediaries would have no incentives to learn about the shock. Nonetheless, assume they were exogenously endowed with such information. At $t = 0$, competitive intermediaries would still bid up to the asset's resale price, in this case independent on the shock. Sellers would not be able to make inference from such bids, and therefore would still behave *as if* selling in a direct market when the shock equals it expected value.

Remark 8. When the shock only enters the sellers' valuation (Appendix A.3), all results of the paper continue to hold irrespective of intermediaries' information over the future realization of the shock.

If the shock affects the buyers' valuation, the extension has less obvious implications. Because the shock determines the price at $t = 1$, intermediaries would profit from learning about its realization at $t = 0$, and only outbid their competitors when the asset's resale price exceeds the expectation. A situation whereby all intermediaries fully predict the shock would paradoxically eliminate the value of the intermediated market. More realistically, however, intermediaries would receive imperfect signals about the shocks' realization. If all intermediaries received the same signal, their bids would reveal it, leading to a situation where sellers behave *as if* selling in a direct market when the shock equals its expected value conditional on the signal. The main result of Theorem 1, that is the fact that the intermediated market is superior under ($\star$), derives from an application of Jensen's Inequality and hence it would still hold since the intermediated market would replicates a direct market where the distribution of the shock is a mean-preserving contraction of the original distribution F.

Remark 9. Assume that intermediaries receive a common signal at $t = 0$ about the shock. If the shock only enters the buyers' valuation, $c_1 < \underline{c}$ (no unraveling) and ($\star$) holds, the intermediated market dominates the direct market.

A model where intermediaries observe private signals of the shock would be more complex to analyze, because the way that bidders' signals aggregate generally depend on the specific market protocol. While a thorough characterization of the enriched model with informed intermediaries is beyond the scope of this paper, I expect the main mechanism to continue to operate, albeit in a more intricate manner.

A.3. General shocks to gains from trade

This section presents a more general specification allowing both the sellers' and the buyers' valuation to contain a random component. As it turns out, no generality is lost in the model presented in the main text, where only buyers' demand is stochastic. As a by-product of this robustness exercise, the analysis of the case in which the sellers' valuation is the only source of uncertainty in gains from trade offers an alternative interpretation of the model: sellers have liquidity needs that are composed of an idiosyncratic component (their type x) and a common component that realizes at $t = 1$. Note that, within this specification, the mechanism leading to the paper's main result is compatible with the view that intermediaries are better suited to hold assets when sellers have high liquidity needs—a fact that, per se, means they can mechanically add value by providing liquidity.

General case. Assume sellers' value for the asset is:

$$U^s(\theta, x, s) = \begin{cases} u(x) + s & \text{if} \quad \theta = g \\ 0 & \text{otherwise} \end{cases}.$$

and the buyers' value is

$$U^b(\theta, c) = \begin{cases} v + b & \text{if} \quad \theta = g \\ b & \text{otherwise} \end{cases}.$$

Assume that both s and b are positive random variables distributed according to some joint measure $H(s, b)$. As in the baseline setup, the assumption that sellers' have no private value for keeping the bad quality asset helps the exposition: it implies that, in the first best, the bad assets are allocated to the buyers, which also occurs in equilibrium under both the direct as well as the intermediated market. Thus, any inefficiency pertains to the allocation of good quality assets.

In the direct market, s and b are observed at the stage where sellers' receive the buyers' offer. Buyers' anticipate that the marginal seller in state (s, b) is indifferent between accepting the offer or retaining and

[28] If the tails decay according to a power law, the ratios $\{F(c)[1 - F(c)]\}/f(c)$ and $F(c)/f(c)$ can be made arbitrarily large at the two extremes of the support of F.

get $u(\hat{x}) + s$. Thus:

$$u(\hat{x}(s,b)) + s = \vartheta(\hat{x}(s,b)) + b.$$

Observe that the marginal seller satisfies:

$$\hat{x}(s,b) = x^*(b-s)$$

where $x^*(.)$ is the marginal seller as a function of the shock in the direct market presented in the main text. This observation suggests that the model in the main text could be interpreted as one where the shock has the role of the random variable $z := b - s$ in the more general setup described here.

Indeed, one can show that, in the intermediated market, where intermediaries make offers before s and b realize, the marginal seller, $\hat{X}$, satisfies: $\hat{X} = \hat{x}(\mathbb{E}s, \mathbb{E}b) = x^*(\mathbb{E}(b-s))$. Finally, aggregating surplus in equilibrium in the direct market, which is a function of the realized b and s and is denoted $\tilde{S}_D(s,b)$, gives:

$$\begin{aligned}\tilde{S}_D(s,b) &= \lambda\left[\int_{\hat{x}(s,b)}^1 (u(x)+s)\,dx + \int_0^{\hat{x}(s,b)} (v+b)\,dx\right] + (1-\lambda)\,b \\ &= \lambda\left[\int_{\hat{x}(s,b)}^1 u(x)\,dx + \int_0^{\hat{x}(s,b)} v\,dx + \hat{x}(s,b)(b-s) + s\right] + (1-\lambda)\,b \\ &= \lambda\left[\int_{x^*(b-s)}^1 u(x)\,dx + \int_0^{x^*(b-s)} (v+b-s)\,dx\right] + (1-\lambda)(b-s) + s \\ &= S_D(b-s) + s.\end{aligned}$$

Similarly, one can prove: $\mathbb{E}\tilde{S}_I(s,b) = \tilde{S}_D(\mathbb{E}s, \mathbb{E}b) = S_D(\mathbb{E}(b-s)) + \mathbb{E}s$. It then follows that:

$$\mathbb{E}\tilde{S}_I(s,b) > \mathbb{E}\tilde{S}_D(s,b) \iff S_D(\mathbb{E}(b-s)) > \mathbb{E}S_D(b-s).$$

In words, the intermediated market dominates under the same conditions identified in Theorem 1, with the distribution of the shock following the distribution of the random variable: $b - s$.

When only sellers' value is stochastic. One interesting case to consider is when b is constant and normalized to zero, so that uncertainty in gains from trade results entirely from fluctuations in sellers' valuation. Note that the largest adverse selection discount is associated to the case where s is large, that is, when sellers have high valuation.

Within this environment, consider the extension with positive intermediary's valuation. The condition in Proposition 10 that identifies the threshold for u^i below which all results of the baseline model continue to hold now reads:

$$u^i \leq \frac{x^*(-\mathbb{E}s)\,\lambda}{x^*(-\mathbb{E}s)\,\lambda + 1 - \lambda}\,v.$$

In principle, the condition is compatible with a situation whereby intermediaries value the asset more than *all* sellers in periods where their valuations are low, that is liquidity needs are high (when s is low). What is crucial to make the intermediated market alleviate adverse selection is that intermediaries value the asset less than (at least some) sellers in periods when liquidity needs are low (s is high). The reason is that in those periods, sellers with low intrinsic preference for the asset (low x) would face a large adverse selection discount, impeding efficient trade.

A.4. Search framework with non-anonymous trading

I adapt here the setup introduced in Section 5 to the case in which buyers can distinguish between intermediaries and original sellers. In this model, the sellers' decision at $t = 0$ whether or not to sell to the intermediary can convey a signal about the seller's type (x, θ) that will enter buyers' beliefs at $t = 1$.

A.4.1. Equilibrium with intermediaries

When intermediaries exist, they are active in two types of equilibria: a fully intermediated pooling equilibrium in which sellers either accept the intermediary's offer or retain the asset, and a semi-separating equilibrium in which only some sellers with good assets and sufficiently high valuation choose to reach out to buyers, and in this way they signal the quality of their assets.

Notation. Formally, denote with $\beta_i(c)$ and $\beta_s(c)$ the buyers' beliefs that the asset available for trade is of high quality, when the realization of the shock is c, and when they face an intermediary and a seller respectively. In a Perfect Bayesian Equilibrium, $\beta_i(c)$ and $\beta_s(c)$ are computed using Bayes' rule on the equilibrium path. The prices that buyers offer are denoted with $p_i^b(c)$ and $p_s^b(c)$ and will again make buyers break-even given their beliefs.

Pooling equilibria. Conjecture a pooling equilibrium in which no sellers with bad assets sell directly to final buyers. Because in equilibrium buyers would not face any seller, the concept of Perfect Bayesian Equilibrium places no restriction on $\beta_s(c)$. To support the conjectured equilibrium, impose that $\beta_s(c) = 0$ for all c, so that $p_s^b(c) = c$ for all c. By reaching out to final buyers, a seller with a good asset and type x receives:

$$\mathbb{E}\max\{c, u(x)\} - \psi$$

and a seller with a bad asset receives:

$$\mathbb{E}c - \psi.$$

Denote with $\tilde{x}$ the seller who is indifferent between selling to the intermediary and retaining the asset until the end of $t = 1$. In the conjectured equilibrium, buyers anticipate that only sellers with bad assets and good assets but valuation below $\tilde{x}$ sell to the intermediary. Thus, $p_i^b(c) = \beta_i(c) + c = \vartheta(\tilde{x}) + c$. Since intermediaries bid up to the asset's expected resale value, it must then be that $\tilde{x} = x^*(\mathbb{E}c)$. That is, the conjectured equilibrium replicates the allocation of the intermediated market described in Section 2. In order for sellers with a bad asset not to be willing to deviate, it must be that:

$$\mathbb{E}p_i^b(c) \geq \mathbb{E}c - \psi$$

which holds in the conjectured equilibrium. In order for sellers with a good asset and type $x \leq x^*(\mathbb{E}c)$ not to be willing to deviate, it must be that:

$$\mathbb{E}p_i^b(c) \geq \mathbb{E}\max\{c, u(x)\} - \psi$$

for all $x \leq x^*(\mathbb{E}c)$. Observe that $\mathbb{E}p_i^b(c) = \vartheta(x^*(\mathbb{E}c)) + \mathbb{E}c$. The condition above is trivially satisfied for a sufficiently large ψ. If $\psi = 0$, one way to satisfy the condition is to assume that $u(x) > c$ for all c and x. Finally, for sellers with a good asset and type $x \geq x^*(\mathbb{E}c)$ not to be willing to deviate, it must be that:

$$u(x) \geq \mathbb{E}\max\{c, u(x)\} - \psi$$

for all $x \geq x^*(\mathbb{E}c)$. The condition is trivially satisfied for a sufficiently large ψ. If $\psi = 0$, one way to satisfy the condition is to assume that $u(x) > c$ for all c and x.

The pooling equilibrium above is supported by the belief that $\beta_s(c) = 0$ for all c. Notice that for types $\theta = b$, when ψ is sufficiently low, the deviation to reaching out to buyers directly is not equilibrium denominated: under sufficiently favorable beliefs, they would deviate. Thus, the pooling equilibrium resists the "*intuitive criterion*" refinement, and could only be ruled out under much more stringent refinements such as D1 (Cho and Kreps, 1987).

Semi-separating equilibria. Conjecture a semi-separating equilibrium in which all sellers with bad assets sell to intermediaries, and sellers owning good assets sell to intermediaries if and only if their type x is below a threshold denoted $\tilde{x}$. To ease the exposition, assume that search costs ψ are such that for all $x \geq \tilde{x}$, sellers of good assets prefer to search rather than retain the asset at $t = 0$. Denote with $\tilde{c}$ the realization of the shock that makes type $\tilde{x}$ indifferent between selling or retaining its good asset, given that its strategy to reach out to buyers perfectly reveals its type. That is, $\tilde{c} = u(\tilde{x}) - v$. In the conjectured equilibrium, it holds that:

$$\beta_s(c)=\begin{cases}\underline{\beta} & \text{if} \quad c\le\tilde{c}\\ 1 & \text{otherwise}\end{cases}$$

and that

$$\beta_i(c)=\frac{\lambda\tilde{x}}{\lambda\tilde{x}+1-\lambda}\qquad\forall c.$$

Since only sellers with good assets and valuation above $u(\tilde{x})$ reach out to buyers at $t=1$, and since buyers price offer cannot exceed $v+c$, no seller accepts the buyers' offer at $t=1$ when $c\le\tilde{c}$. Therefore, the concept of Perfect Bayesian Equilibrium does not place any restriction on the belief $\underline{\beta}$. For any given $\underline{\beta}$, type $\tilde{x}$ is indifferent between searching and selling to the intermediary at $t=0$ if the following holds:

$$\mathbb{E}p_i^b(c)=\mathbb{E}\max\left\{p_s(c),u(\tilde{x})\right\}-\psi$$

which expanded gives:

$$\vartheta(\tilde{x})+\mathbb{E}c=\int_{\underline{c}}^{\tilde{c}}u(\tilde{x})\,dF(c)+\int_{\tilde{c}}^{\overline{c}}v+c\,dF(c)-\psi. \tag{A.2}$$

Clearly, if $(A.2)$ holds, then all sellers with good assets and type below $\tilde{x}$ would strictly prefer to sell to the intermediary. The same holds for a seller with a bad asset. Note that equation $(A.2)$ pins down a unique value for $\tilde{x}$ for any given ψ, and observe that $\tilde{x}$ decreases with ψ. Finally, observe that the construction of the semi-separating equilibrium does not rely on a particular choice of the off-equilibrium belief $\underline{\beta}$, and therefore that the equilibrium would survive both the intuitive criterion and the D1 refinement.

To have some separation across types, search must be sufficiently costly. Indeed, consider a candidate fully separating equilibrium in which sellers reach out to buyers if and only if they own good quality assets, and sell to the intermediary otherwise. In this case, the condition that ensures no seller with a good asset deviates is:

$$\int_{\underline{c}}^{\max\left\{\underline{c},u(0)-v\right\}}u(0)\,dF(c)+\int_{\max\left\{\underline{c},u(0)-v\right\}}^{\overline{c}}v+c\,dF(c)-\psi\ge\max\left\{\mathbb{E}c,u(0)\right\}$$

and the condition that ensures no seller with a bad asset deviates is:

$$\mathbb{E}c\ge\int_{\underline{c}}^{\max\left\{\underline{c},u(0)-v\right\}}\underline{\beta}v+c\,dF(c)+\int_{\max\left\{\underline{c},u(0)-v\right\}}^{\overline{c}}v+c\,dF(c)-\psi.$$

Note that if ψ falls below the critical value $\underline{\psi}:=[1-F(u(0)-v)]\,v$, it is not possible to sustain an equilibrium in which some sellers of good assets separate by searching, because sellers with bad assets would deviate, despite the lowest possible belief on asset quality is imposed for all states in which $c\le\tilde{c}$, that is when no seller with a good asset is expected to accept the buyers' offer in equilibrium.

The role of search costs. What would occur below the threshold $\underline{\psi}$? Evidently, as it also happens in the version of the model with anonymous trades, sufficiently low search costs are also compatible with an equilibrium in which no intermediaries are active. Without equilibrium selection criteria, it is not possible to derive a sharp prediction. If one assumes that for low search costs agents would coordinate on the pooling equilibrium in which all sellers search for buyers, the model would again admit a parameter region in which welfare is increasing in search costs.

Remark 10 (*Role of search costs*)**.** In the model with intermediaries, total welfare may be non-monotone in search costs.

In this case, a reduction in search costs may harm welfare when it is so large to prevent sellers to signal their higher quality via costly search.

A.4.2. Equilibrium without intermediaries

Consider now an environment in which intermediaries are not present. The sellers' strategy space at $t=0$ reduces to a binary choice: either pay the search cost to reach out to buyers at $t=1$, or retain the asset. In this setup, the distinction between anonymous and non-anonymous trading is irrelevant, since buyers can only face sellers at $t=1$. The setup thus coincides with what analyzed in Section 5 in the case in which there are no intermediaries. In the proof of Theorem 2, I characterize the unique equilibrium of this game. The equilibrium features all sellers with bad assets and sellers with good assets and valuation below a threshold $\tilde{X}$ to search for buyers at $t=1$, and all other sellers to retain the asset. That is, in equilibrium, *costly search does not signal higher quality*. The threshold is found by solving:

$$u\left(\tilde{X}\right)=\mathbb{E}\max\left\{\vartheta\left(\tilde{X}\right)+c,u\left(\tilde{X}\right)\right\}-\psi \tag{A.3}$$

Fixing ψ, the threshold above which sellers separate in the equilibrium with intermediaries is below the threshold above which sellers retain in the equilibrium without intermediaries. That is, $\tilde{X}>\tilde{x}$ where $\tilde{x}$ solves equation $(A.3)$. For sufficiently low values of ψ, $\tilde{X}=1$, in which case the equilibrium allocation coincides with that implied by the non-intermediated market analyzed in Section 2.

One immediate implication is that, compared to the semi-separating equilibrium constructed in the model with intermediaries, adverse selection here is stronger in states in which c is low, since no seller with good assets is committed not to retain the asset at $t=1$ upon observing a low offer. This mechanism is the same that operates in the model analyzed in the main text. On top of that, the semi-separating equilibrium also features lower adverse selection in states in which c is high, since all good asset owners who choose to search can sell the asset at the efficient, full information price.

In Theorem 2, I show that, for any ψ, welfare is lower in this setup compared to one in which intermediaries are present but trade is anonymous. Since non-anonymous trading also permits good asset owners to signal their quality, provided that intermediaries are present, the next remark follows.

Remark 11 (*Role of intermediaries*)**.** For any level of search costs, the presence of intermediaries increases welfare.

Absent the intermediary, a seller who does not search ends up retaining the asset until the end of period $t=1$. Since retention is more valuable for sellers owning good quality assets, there is no scope for signaling. When instead intermediaries are present at $t=0$, sellers with low valuation can avoid costly search, while all others endure the search cost, maintaining the option to retain in case the buyers' offer is low. By doing so, they separate.

Appendix B. Proofs

Preliminaries

I prove here that Definition 1 implies that, in the non-intermediated market, the equilibrium is unique, and the marginal seller of a good asset is either zero, one, or the highest interior solution to equation (2). The price that buyers bid up to is always strictly positive, hence all bad asset owners sell in equilibrium. For any best price offer p, a seller with a good asset accepts the offer if and only if $p\ge u(x)$. Since $u(x)$ is strictly increasing, the good asset owner's strategy is characterized, for every price offer p, by the function $x(p)$:

$$x(p)=\begin{cases}0 & \text{if} \quad p<u(0)\\ u^{-1}(p) & \text{if} \quad u(0)\le p<u(1).\\ 1 & \text{otherwise.}\end{cases}$$

I consider three cases separately.

Case I. Assume $u(1) < \lambda v + c$. In this case, it must be that in equilibrium all sellers owning a good asset choose to sell it. In anticipation of a contradiction, suppose this is not the case. Since sellers follow a cutoff strategy, denote $\tilde{x}$ the marginal seller. By definition, it holds that: $p = u(\tilde{x})$. Recall the definition of the function:

$$\vartheta(x) := \frac{\lambda x}{\lambda x + 1 - \lambda} v$$

and observe that $p = \vartheta(\tilde{x}) + c$, due to buyers' break-even condition. Note that $\vartheta(1) = \lambda v$ and thus $p' = \lambda v + c > p$. Because $u(1) < \lambda v + c$, all sellers are willing to sell at p'. There is a price strictly higher than p that makes buyers break-even and is also consistent with the sellers' optimal response, leading to a contradiction.

Case II. Assume $u(1) > \lambda v + c$, but at least one solution exists to $\vartheta(x) + c = u(x)$. Then, the same argument implies that the equilibrium marginal seller is given by the highest such solution.

Case III. Finally, assume that for all x it holds that $u(x) > \vartheta(x) + c$. Then it is immediate to observe that in equilibrium all good asset holders retain the asset.

The intermediated market can be analyzed similarly. Definition 1 guarantees that the equilibrium is unique, and, when the marginal seller is interior, it is characterized by the highest solution to the equation

$$\vartheta(\hat{x}) + \mathbb{E}c = u(\hat{x}). \tag{B.1}$$

Examples in which the conditions of Theorem 1 hold

Example 3. The distribution of sellers' valuations is exponential, that is $g(u) = \alpha e^{-\alpha u}$ for $u \geq 0$. Then, the function $u(x)$ is $u(x) = -1/\alpha \ln(1-x)$. One way to ensure that Assumption 3 hold is to impose the lower bound on $u'(x)$ is above the upper bound on ϑ', giving $\alpha < 1/\lambda v$. The sufficient condition for ($\star$) to hold, $g' \leq -g/(1-\lambda)v$, implies $\alpha > 1/(1-\lambda)v$. The fact that $u(0) = 0$ and $u(1)$ tends to ∞ implies that the equilibrium is always interior, that is $\underline{c} > c_1$. Hence, for $1/(1-\lambda)v < \alpha < 1/\lambda v$ the intermediated market dominates.

Example 4. The fact that ($\star$) must only hold in the region where $x \in \chi$ aids the search for a case where the condition is met. Consider the Kumaraswamy distribution with parameters $(2, 4)$, which has a single-peaked density $g(u) = 8u\left(1-u^2\right)^3$ defined in the domain $u \in (0,1)$. For low values of u, it holds that $g' > 0$. Moreover, there exists a unique $\hat{u}$ such that $g'(u) \leq -g(u)/(1-\lambda)v$ if and only if $u \geq \hat{u}$.[29] Recall that the utility of the marginal seller is at least $\underline{c}$. Setting $\lambda = 1/8$ and $v = 1$, verify that $g'(u) \leq -g(u)/(1-\lambda)v$ holds at $u = 1/2$. Therefore, by setting $\underline{c} = 1/2$, one can ensure that, for all c, the interior solutions $x^*(c)$, when they exist, lie in the region where ($\star$) holds. To make sure that the marginal seller $x^*(c)$ is, in fact, always interior, set $\bar{c}$ so that $u(1) = 1 > \lambda v + \bar{c}$.

Proof of Lemma 1. Observe that since $x(c)$ is the highest solution to (2), then, for all $x > x(c)$, it must be that $\vartheta(x) + c > u(x)$. Since both $\vartheta(x)$ and $u(x)$ are continuous and differentiable, it must then hold that, $\vartheta'(x(c)) < u'(x(c))$. Using the Implicit Function Theorem, one obtains:

$$x'(c) = \left[u'(x(c)) - \vartheta'(x(c))\right]^{-1}$$

proving the result. ■

Proof of Proposition 1. To characterize the equilibrium function $x^*(c)$ for all positive c, distinguish three cases.

Case I. Assume a $y \in (0,1)$ exists such that $u'(y) = \vartheta'(y)$, and hence, by Assumption 3, $u'(x) > (<) \vartheta'(x)$ for all $x > (<) y$. Here, I consider two subcases. First assume $u(y) > \vartheta(y)$.

Then, there must exist a positive value of c, denoted c_1, that solves,

$$u(y) = \vartheta(y) + c_1.$$

Thus, y solves (2) at c_1. Since $u'(x) > \vartheta'(x)$ for all $x \geq y$, it also holds that $u(1) > \vartheta(1)$, therefore there must exist a c, denoted c_2 such that

$$u(1) = \vartheta(1) + c_2.$$

Note that, in the interval $\left(c_1, c_2\right)$, equation (2) cannot admit more than one solution in $[y,1]$ by monotonicity of $u(x) - \vartheta(x)$. Moreover, a solution in $[y,1]$ exists and, since $u(1) < \vartheta(1) + c = \lambda v + c$, it therefore identifies the equilibrium $x^*(c)$ as proven in the previous paragraphs. Now, I can prove $x^*(c)$ is continuous in $\left(c_1, c_2\right)$. To find a contradiction, assume that it is not, so that there exists a positive ε and a sequence $h_n \to 0$ as $n \to \infty$, such that $|x^*(c + h_n) - x^*(c)| \geq \varepsilon$ for every $n \in \mathbb{N}$. Note that since $x^*(c) \in [y,1]$ for all $c \in \left(c_1, c_2\right)$, a subsequence h_{n_k} would also exist such that $x^*\left(c + h_{n_k}\right)$ converges to some $z \in [y,1]$ and $z \neq x^*(c)$ for every $k \in \mathbb{N}$. Since $x^*\left(c + h_{n_k}\right)$ is the solution to (2) at $c + h_{n_k}$, then $u\left(x^*\left(c + h_{n_k}\right)\right) - \vartheta\left(x^*\left(c + h_{n_k}\right)\right) - c - h_{n_k} = 0$. Continuity of $H(x,c) := u(x) - \vartheta(x) - c$ would imply that $H\left(x^*\left(c + h_{n_k}\right), c + h_{n_k}\right)$ converges to $H(z,c)$ and $H(z,c) = 0$, which means z also solves (2) at c, but is different from $x^*(c)$, a contradiction.

Since $x^*\left(c_2\right) = 1$ solves (2) by construction, and the marginal seller is of type 1 for all $c > c_2$, the equilibrium $x^*(c)$ is continuous for all values of $c > c_1$.

Recall that, for all $x < y$, it holds that $u'(x) < \vartheta'(x)$. Since $u(y) = \vartheta(y) + c_1$, $u(x) > \vartheta(x) + c$ for all x and all $c < c_1$. Thus, every seller retains, that is, $x^*(c) = 0$ for all $c < c_1$. Since, $y \in (0,1)$, $x^*\left(c_1\right) = y > 0$, hence there is a discontinuity.

Assume now that $u(y) < \vartheta(y)$. Then, the c_1 that solves $u(y) = \vartheta(y) + c_1$ is negative. Since $u'(x) > \vartheta'(x)$ for all $x \geq y$, the analysis above can be repeated, implying $x^*(c)$ is continuous (and strictly positive) for all positive c.

Case II. Assume now $u'(x) > \vartheta'(x)$ for every x. Consider again two subcases. If $u(1) < \vartheta(1)$, as before $x^*(c) = 1$ for every c. If instead $u(1) > \vartheta(1)$, then there exists a c_2 such that $u(1) = \vartheta(1) + c_2$. Note a c_1 such that $u(0) = \vartheta(0) + c_1 = c_1$ always exists and use the arguments above to prove that $x^*(c) = 1$ for all $c \geq c_2$ and is continuous for all values of $c > c_1$. Moreover, $u(x) > \vartheta(x) + c$ for all $c < c_1$ and for every x, thus every seller retains, that is, $x^*(c) = 0$ and therefore $x^*(c)$ is continuous everywhere.

Case III. Assume now $u'(x) < \vartheta'(x)$ for every x. As before, if $u(1) < \vartheta(1)$, $x^*(c) = 1$ for every c. If instead $u(1) > \vartheta(1)$, again there exists a c_2 such that $u(1) = \vartheta(1) + c_2$, so $x^*(c) = 1$ for all $c \geq c_2$. However, for all $c < c_2$, it would hold that $u(x) > \vartheta(x) + c$ for all x, thus $x^*(c) = 0$. ■

Fig. B.5 shows how an equilibrium $x^*(c)$ is found and illustrates the role of Assumption 3.

Proof of Lemma 2. Lemma 2 follows immediately from the equilibrium definition and the observation that intermediaries accept any non-negative best price offer. ■

Proof of Proposition 2. For Proposition 2, the same arguments used to derive the statements in Proposition 1 can be applied, noting that equation $(B.1)$ replaces condition (2). ■

Proof of Theorem 1. The proof uses the fact, implied by Proposition 2, that $X^* = x^*(\mathbb{E}c)$.

Part I. To prove (I), it suffices to write expected surplus in the two alternative market setups. Since $\mathbb{E}c \geq c_2$, use $X^* = 1$. The intermediated

[29] The cutoff is the unique solution to $7u^2 - 1 = u\left(1-u^2\right)\left[(1-\lambda)v\right]^{-1}$.

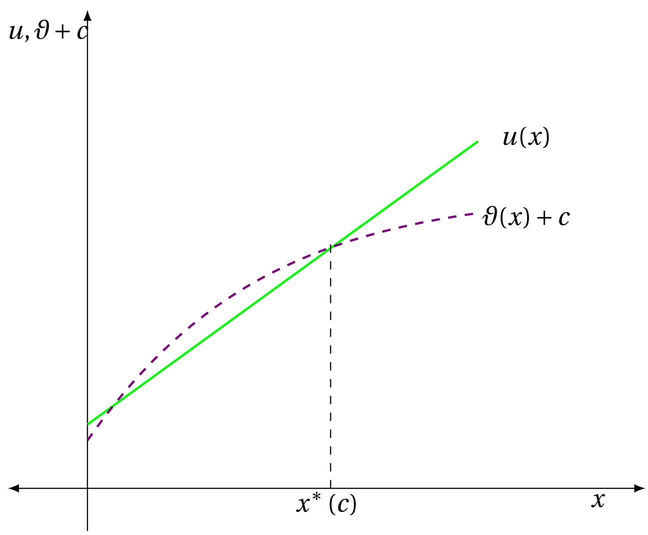

Fig. B.5. The green (solid) line represents $u(x)$, which is linear, indicating uniformly distributed valuations. The violet (dashed) curve is the break-even price $\vartheta(x) + c$ for every marginal seller x, for a given realization of c. There are two intersections and the equilibrium $x^*(c)$ is the highest of the two. Note that $u(x)$ is flatter than $\vartheta(x)$ only if x is low, and steeper only if it is high (Assumption 3 holds). An higher realization of c shifts the violet curve upwards until an intersection occurs at $x = 1$. This occurs at $c = c_2$. For all c above c_2, $x^*(c) = 1$. A lower realization of c shifts the violet curve downwards, until an intersection occurs at the unique x where the two functions have the same slope. This occurs at $c = c_1$, and in this case, $x^*(c_1) > 0$. Note that, for all c below c_1, $x^*(c) = 0$. Thus, $x^*(c)$ exhibits only one discontinuity, around which the market for good assets unravels. If instead $u'(x) > \vartheta'(x)$ for all x, $x^*(c)$ is continuous everywhere.

market generates higher expected surplus if and only if:

$$\mathbb{E}\left\{(1-\lambda)c + \lambda\int_0^1 (v+c)\,dx\right\}$$
$$\geq \mathbb{E}\left\{(1-\lambda)c + \lambda\left[\int_0^{x^*(c)} (v+c)\,dx + \int_{x^*(c)}^1 u(x)\,dx\right]\right\}$$

giving

$$\mathbb{E}\left[\int_{x^*(c)}^1 (v + c - u(x))\,dx\right] \geq 0$$

which is implied by the condition in statement (I).

Part II, i. To prove (II), note that $\mathbb{E}S_I(c) = S_D(\mathbb{E}c)$. Therefore, the comparison between the expected value of $S_I(c)$ and of $S_D(c)$ boils down to an application of Jensen's inequality.

In particular, focusing on the subcase where $c \in [c_1, c_2]$, note that the function $S_D(c)$ is continuous in its entire domain, since $x^*(c)$ is continuous in such a region by Proposition 1. The second derivative of $S_D(c)$ is:

$$S''_D(c) = \lambda\left\{x^{*\varepsilon}(c)\left[v + c - u\left(x^*(c)\right)\right] + 2x^{*'}(c) - x^{*'}(c)^2 u'\left(x^*(c)\right)\right\}.$$

Using the Implicit Function Theorem, replace

$$x^{*'}(c) = \left[u'(x(c)) - \vartheta'(x(c))\right]^{-1}$$

and again differentiating:

$$x^{*''}(c) = \left[u'(x(c)) - \vartheta'(x(c))\right]^{-3}\left[\vartheta''(x(c)) - u''(x(c))\right].$$

From the equilibrium condition (2), obtain $v+c-u(x^*(c)) = v-\vartheta(x^*(c))$. Using the fact that $u'(x^*(c)) > \vartheta'(x^*(c))$, replacing and collecting all terms gives condition ($\star$) evaluated at $x^*(c)$. Since $S_D(c)$ is concave, then, for all prior distributions of the shock, $\mathbb{E}S_I(c) \geq \mathbb{E}S_D(c)$.

Part II, ii. The case where $c_1 > \underline{c}$ requires additional analysis, since one can verify that, for $c < c_1$, $S_D(c)$ is linear, increasing, with slope $S'_D(c) = 1 - \lambda < \lim_{c \searrow c_1} S'_D(c)$. Moreover, $S_D(c)$ is generically discontinuous at c_1. Despite the function $S_D(c)$ not being concave in its entire domain, it is still possible to apply Jensen's inequality provided one can prove that the tangent of $S_D(c)$ at $c = \mathbb{E}c$ lies above the function $S_D(c)$, for all values of c. Essentially this means the function $-S_D(c)$ is subdifferentiable at $c = \mathbb{E}c$, a general property that holds in the entire domain for all convex functions, although not necessarily differentiable, and that is sufficient to prove Jensen's inequality. In the region $[\underline{c}, c_1)$ it holds that $x^*(c) = 0$. Therefore, for the tangent at $c = \mathbb{E}c$ to lie above $S_D(c)$ for all c in $[\underline{c}, c_1)$, it suffices to check that this holds at $\underline{c}$. One then obtains:

$$\int_0^{x^*(\mathbb{E}c)} [v + \mathbb{E}c - u(x)]\,dx \geq (\mathbb{E}c - \underline{c})\left\{x^{*'}(\mathbb{E}c)\left[v + \mathbb{E}c - u(x^*(\mathbb{E}c))\right] + x^*(\mathbb{E}c)\right\}.$$

Recall that $x^{*'}(\mathbb{E}c) = \left[u'(x^*(\mathbb{E}c)) - \vartheta'(x^*(\mathbb{E}c))\right]^{-1}$, and note that, since ($\star$) holds, the right-hand side of the condition above is a decreasing function of the variable $y := x^*(\mathbb{E}c)$. Moreover, since $v + \mathbb{E}c - u(x^*(\mathbb{E}c)) > 0$, a lower bound on the left-hand side is obtained by replacing $x^*(\mathbb{E}c)$ with its lower bound. To obtain a sufficient condition expressed in terms of model's primitives, I can thus replace the solution $x^*(\mathbb{E}c)$ with its lower bound, which is $\underline{x}(\mathbb{E}c)$. This substitution yields condition ($\star\star$). ■

Proof of Proposition 3. Define the value of intermediation as:

$$V := \mathbb{E}S_I(c) - \mathbb{E}S_D(c).$$

The goal of the proof is to show that a perturbation of $u(x)$ in the direction of $p(x)$, for a function $p(x) > 0$ with $p'(x) < 0$ and $p''(x) = 0$, increases V. This is equivalent to proving:

$$\frac{\partial \mathbb{E}S_I}{\partial u}\cdot p > \frac{\partial \mathbb{E}S_D}{\partial u}\cdot p.$$

Note:

$$\begin{aligned}\frac{\partial \mathbb{E}S_D}{\partial u}\cdot p &= \lim_{\varepsilon\to 0}\frac{\int_c S_D(c;u+\varepsilon p)dF(c) - \int_c S_D(c;u+\varepsilon p)dF(c)}{\varepsilon}\\ &= \int_c\left\{\lim_{\varepsilon\to 0}\frac{S_D(c;u+\varepsilon p) - \int_c S_D(c;u+\varepsilon p)}{\varepsilon}\right\}dF(c)\\ &= \mathbb{E}\frac{\partial S_D}{\partial u}\cdot p\end{aligned}$$

where I have used the definition of Gateaux derivative and the linearity of the expectation operator. One can similarly show

$$\frac{\partial \mathbb{E}S_I}{\partial u}\cdot p = \mathbb{E}\frac{\partial S_I}{\partial u}\cdot p.$$

Recall that $\mathbb{E}S_I(c) = S_D(\mathbb{E}c)$. Thus, it also holds that: $\mathbb{E}\frac{\partial S_I}{\partial u}\cdot p = \frac{\partial S_D(\mathbb{E}c)}{\partial u}\cdot p$. Therefore, the goal of the proof is to show:

$$\frac{\partial S_D(\mathbb{E}c)}{\partial u}\cdot p > \mathbb{E}\frac{\partial S_D}{\partial u}\cdot p$$

which, by Jensen's inequality, is true if the function $\eta(c) := \frac{\partial S_D}{\partial u}\cdot p$ is concave.

Note also, that

$$\frac{\partial S_D}{\partial u}\cdot p = \lim_{\varepsilon\to 0}\frac{S_D(c;u+\varepsilon p) - S_D(c;u)}{\varepsilon} = \frac{\partial S_D(c;u+\varepsilon p)}{\partial \varepsilon}\Big|_{\varepsilon=0}.$$

The function $\eta(c)$ is thus:

$$\eta(c) = \lambda\left\{x_p^*(c;u)\left[v - \vartheta\left(x^*(c;u)\right)\right] + \int_{x(c;u)}^1 p(x)\,dx\right\}.$$

where $x_p^*(c;u)$ is the Gateaux derivative of x^* with respect to u in the direction of p, and is computed using the Implicit Function Theorem as:

$$x_p^*(c;u) = -\frac{p(x^*(c;u))}{u_x(x^*(c;u);u) - \vartheta_x(x^*(c;u))}.$$

The first derivative of $\eta(c)$ is:

$$\eta'(c) = \lambda \left\{ \begin{array}{c} x^*_{pc}(c;u)\left[v - \vartheta\left(x^*(c;u)\right)\right] - x^*_p(c;u)\, x^*_c(c;u)\, \vartheta_x\left(x^*(c;u)\right) \\ x^*_c(c;u)\, p\left(x^*_c(c;u)\right) \end{array} \right\}$$

where $x^*_c(c;u)$ is the first derivative of $x^*(c;u)$ with respect to c, and is derived in the previous proof using the Implicit Function Theorem. The function $x^*_{pc}(c;u)$ is the Gateaux derivative of x^*_c with respect to u in the direction of p, and is derived using the Implicit Function Theorem as:

$$x^*_{pc}(c;u) = \frac{u_{xx}\left(x^*(c;u);u\right) - \vartheta_{xx}\left(x^*(c;u)\right)}{\left[u_x\left(x^*(c;u);u\right) - \vartheta_x\left(x^*(c;u)\right)\right]^3} p\left(x^*(c;u)\right) - \frac{p'\left(x^*(c;u)\right)}{\left[u_x\left(x^*(c;u);u\right) - \vartheta_x\left(x^*(c;u)\right)\right]^2}.$$

The second derivative of $\eta(c)$ is:

$$\eta''(c) = \lambda \left\{ \begin{array}{c} x^*_{pcc}(c;u)\left[v - \vartheta\left(x^*(c;u)\right)\right] - 2x^*_{pc}(c;u)\, x^*_c(c;u)\, \vartheta_x\left(x^*(c;u)\right) \\ -x^*_p(c;u)\, x^*_{cc}(c;u)\, \vartheta_x\left(x^*(c;u)\right) - x^*_p(c;u)\left[x^*_c(c;u)\right]^2 \vartheta_{xx}\left(x^*(c;u)\right) \\ -x^*_{cc}(c;u)\, p\left(x^*(c;u)\right) - \left[x^*_c(c;u)\right]^2 p'\left(x^*(c;u)\right) \end{array} \right\}$$

where $x^*_{cc}(c;u)$ is the first derivative of $x^*_c(c;u)$ with respect to c, and is derived in the previous proof using the Implicit Function Theorem. The function $x^*_{pcc}(c;u)$ is the derivative of x^*_{pc} with respect to c, and is derived using the Implicit Function Theorem as:

$$x^*_{pcc}(c;u) = \left\{ \begin{array}{c} -\frac{3\left[u_{xx}\left(x^*(c;u);u\right) - \vartheta_{xx}\left(x^*(c;u)\right)\right]^2}{\left[u_x(x^*(c;u);u) - \vartheta_x(x^*(c;u))\right]^4} p\left(x^*(c;u)\right) x^*_c(c;u) \\ +\frac{\left[u_{xxx}\left(x^*(c;u);u\right) - \vartheta_{xxx}\left(x^*(c;u)\right)\right]}{\left[u_x(x^*(c;u);u) - \vartheta_x(x^*(c;u))\right]^3} p\left(x^*(c;u)\right) x^*_c(c;u) \\ \frac{2\left[u_{xx}\left(x^*(c;u);u\right) - \vartheta_{xx}\left(x^*(c;u)\right)\right]}{\left[u_x(x^*(c;u);u) - \vartheta_x(x^*(c;u))\right]^3} p'\left(x^*(c;u)\right) x^*_c(c;u) \end{array} \right\}.$$

Consider first the case where $p'\left(x^*(c;u)\right) = 0$. Then, rearranging terms, $\eta''(c) < 0$ if and only if:

$$\left\{ \frac{3\left[u_{xx}(x^*(c;u);u) - \vartheta_{xx}(x^*(c;u))\right]^2}{\left[u_x(x^*(c;u);u) - \vartheta_x(x^*(c;u))\right]^2} - \frac{\left[u_{xxx}(x^*(c;u);u) - \vartheta_{xxx}(x^*(c;u))\right]}{\left[u_x(x^*(c;u);u) - \vartheta_x(x^*(c;u))\right]} \right\} \left(v - \vartheta\left(x^*(c;u)\right)\right)$$
$$>$$
$$\frac{\left[u_{xx}(x^*(c;u);u) - \vartheta_{xx}(x^*(c;u))\right]\left[u_x(x^*(c;u);u) - 4\vartheta_x(x^*(c;u))\right]}{u_x(x^*(c;u);u) - \vartheta_x(x^*(c;u))} + \vartheta_{xx}\left(x^*(c;u)\right)$$

Assuming $u_{xx}(x) > \vartheta_{xx}(x)$ and $u_{xxx}(x) < \vartheta_{xxx}(x)$ hold everywhere, and noting $\vartheta_{xx}(x) < 0$, it is sufficient for $\eta''(c) < 0$ that:

$$3\left[u_{xx}\left(x^*(c;u);u\right) - \vartheta_{xx}\left(x^*(c;u)\right)\right]\left(v - \vartheta\left(x^*(c;u)\right)\right)$$
$$>$$
$$\left[u_x\left(x^*(c;u);u\right) - \vartheta_x\left(x^*(c;u)\right)\right]\left[u_x\left(x^*(c;u);u\right) - 4\vartheta_x\left(x^*(c;u)\right)\right]$$

which holds at every x by condition ($\star$).

Consider now the case where $p'\left(x^*(c;u)\right) < 0$. The three terms to be added to $\eta''(c)$ are:

$$\frac{2\left[u_{xx}\left(x^*(c;u);u\right) - \vartheta_{xx}\left(x^*(c;u)\right)\right]}{\left[u_x\left(x^*(c;u);u\right) - \vartheta_x\left(x^*(c;u)\right)\right]^3}\left(v - \vartheta\left(x^*(c;u)\right)\right) p'\left(x^*(c;u)\right) x^*_c(c;u),$$

$$\frac{2\vartheta_x\left(x^*(c;u)\right)}{\left[u_x\left(x^*(c;u);u\right) - \vartheta_x\left(x^*(c;u)\right)\right]^2} p'\left(x^*(c;u)\right) x^*_c(c;u)$$

and

$$-p'\left(x^*(c;u)\right)\left[x^*_c(c;u)\right]^2.$$

The sum of the three terms is negative if and only if:

$$2\left[u_{xx}\left(x^*(c;u);u\right) - \vartheta_{xx}\left(x^*(c;u)\right)\right]\left(v - \vartheta\left(x^*(c;u)\right)\right)$$
$$>$$
$$\left[u_x\left(x^*(c;u);u\right) - \vartheta_x\left(x^*(c;u)\right)\right]\left[u_x\left(x^*(c;u);u\right) - 3\vartheta_x\left(x^*(c;u)\right)\right]$$

which holds at every x by condition ($\star$).

Similarly, consider perturbations to $U^b(\theta, c)$ of the form:

$$\tilde{U}^b(\theta, c; q) = U^b(\theta, c) + \varepsilon q(\theta, c).$$

Proposition 3 concerns the case where $q(\theta, c)$ is constant. For concreteness, I here consider the case where q is positive (i.e., an upward shift in buyers' valuation). Aggregate surplus can be rewritten to make explicit its dependence on the function U^b. In the non-intermediated market, it gives:

$$S_D\left(c; U^b\right) = (1-\lambda)\, U^b(\theta = b, c) + \lambda\left[\int_{x^*(c;U^b)}^1 u(x)\, dx + \int_0^{x^*(c;U^b)} U^b(\theta = g, c)\, dx\right].$$

In the intermediated market, surplus is:

$$S_I\left(c; U^b\right) = (1-\lambda)\, U^b(\theta = b, c) + \lambda\left[\int_{\tilde{X}}^1 u(x)\, dx + \int_0^{\tilde{X}} U^b(\theta = g, c)\, dx\right].$$

Recall that $\tilde{X} = x^*\left(\mathbb{E}c; U^b\right)$ and that therefore, $\mathbb{E}\left[S_I\left(c; U^b\right)\right] = S_D\left(\mathbb{E}c; U^b\right)$.

Note $x^*_q\left(c; U^b\right)$ is the Gateaux derivative of x^* with respect to U^b in the direction of q, and it is easy to see that $x^*_q\left(c; U^b\right) = \frac{q}{u_x\left(x^*\left(c;U^b\right)\right) - \vartheta_x\left(x^*\left(c;U^b\right)\right)}$. Moreover,

$$\frac{\partial S_D}{\partial U^b} \cdot q = \lim_{\varepsilon \to 0} \frac{S_D\left(c; U^b + \varepsilon q\right) - S_D\left(c; U^b\right)}{\varepsilon} = \frac{\partial S_D\left(c; U^b + \varepsilon q\right)}{\partial \varepsilon}\Big|_{\varepsilon=0}$$

and therefore:

$$\frac{\partial S_D}{\partial U^b} \cdot q = (1-\lambda)\, q + \lambda\left\{x^*_q\left(c; U^b\right)\left[v - \vartheta\left(x^*\left(c; U^b\right)\right)\right] + x^*\left(c; U^b\right) q\right\}.$$

Denote this function $\gamma(c)$. The first derivative of $\gamma(c)$ is:

$$\gamma'(c) = \lambda \left\{ \begin{array}{c} x^*_{qc}\left(c; U^b\right)\left[v - \vartheta\left(x^*\left(c; U^b\right)\right)\right] - x^*_q\left(c; U^b\right) x^*_c\left(c; U^b\right) \vartheta_x\left(x^*\left(c; U^b\right)\right) \\ +x^*_c\left(c; U^b\right) q \end{array} \right\}.$$

Evidently, the first and second derivatives of $\gamma(c)$ mirror, with opposite signs, those of $\eta(c)$ defined in the first part of the proof. Thus, an equal uniform increase in U^b has the same effect of a uniform decrease in $u(x)$, completing the proof. ■

Proof of Proposition 4. Proposition 4 pertains to specific perturbations of the distribution of the shock that induce mean-preserving spread or contractions to it. The fact that $\tilde{X}$ only depends on $\mathbb{E}c$ and that $S_I(c)$ is linear in c imply that $\mathbb{E}S_I(c)$ is not affected by such perturbations. By contrast, conditions ($\star$) and $c_1 \geq \underline{c}$ imply concavity of $S_D(c)$. Hence, by Jensen's inequality, perturbations that induce mean preserving spreads (contractions) in the distribution of c decrease (increase) $\mathbb{E}S_D(c)$, and thus they increase (decrease) the value of intermediation. ■

Proof of Proposition 5. Set $\psi = 0$, and conjecture an equilibrium in which every seller searches for buyers at $t = 1$. Since searching for buyers entails no costs, no seller strictly profits from deviating and selling to the intermediary at $t = 0$.

Conjecture now an equilibrium in which only sellers with bad assets and owners of good assets and type below $x^*\left(\underline{c}\right)$ accept the intermediaries' offer with positive probability. Clearly, searching for buyers is valuable if and only if the realization of the shock, and hence the buyers' price offer at $t = 1$ can affect the seller's decision whether to trade or retain the asset. Since owners of bad assets and owners of good assets and type below $x^*\left(\underline{c}\right)$ would always sell at $t = 1$, they do not profit from deviating to search.

The statements in part (II) and (III) derive from the full characterization of the equilibrium as a function of search costs, when $\psi > 0$. This characterization is included in the proof of Proposition 6 given below. ■

Proof of Proposition 6. Denote $p(c)$ the equilibrium market price at $t = 1$. Since the buyers' valuation is positive irrespective of the asset's quality, it holds that $p(c) > 0$ for all c. Owners of bad assets sell their assets for any non-negative price at $t = 1$, therefore they do not pay the search cost in equilibrium, and sell to the intermediary at a price equal to the expectation of $p(c)$.

Owners of good asset face the choice between selling the asset to the intermediary at $t = 0$, incur the search cost to observe the buyers' offer, or retain the asset until the end of $t = 1$ without searching. The expected payoff for selling is the expectation of $p(c)$, which is independent on their type x, while the payoff from searching is:

$$\mathbb{E}\max\{p(c), u(x)\} - \psi.$$

Finally, the payoff from retaining the asset is $u(x)$. Clearly, the payoff from retaining increases in x more steeply then the payoff from searching, resulting in a strategy profile that can be described by the partition: $\{0, X_0, X_1, 1\}$. I can now characterize the equilibrium price $p(c)$ that makes buyers break even. To do so, it is useful to define, for any given X, the function $\hat{c}(X)$, denoting the realization of the shock that solves:

$$\vartheta(X) + \hat{c}(X) = u(X).$$

Using the arguments in the proof of Proposition 1, observe that, under assumption A4, a unque $\hat{c}(X)$ exists for every X.

At $t = 1$, all intermediaries accept the buyers' offer. Those seller of good assets who search for buyers accept the buyers' offer if and only if $p(c) \geq u(x)$. Recall that only types in the interval $[X_0, X_1]$ observe the buyers' offer. It follows that, in state c, the measure of good assets available for trade is $\lambda\tilde{X}(c)$ where $\tilde{X}(c)$ is given by:

$$\tilde{X}(c) = \begin{cases} X_0 & \text{if} \quad c < \min\{\underline{c}, \hat{c}(X_0)\} \\ \sup\{x \in [X_0, X_1] : \vartheta(x) + c = u(x)\} & \text{if} \quad \min\{\underline{c}, \hat{c}(X_0)\} \\ & \leq c < \min\{\bar{c}, \hat{c}(X_1)\} \\ X_1 & \text{if} \quad \min\{\bar{c}, \hat{c}(X_1)\} \leq c. \end{cases}$$

The price in equilibrium is then: $p(c) = \vartheta(\tilde{X}(c)) + c$.

If the threshold X_0 is interior, it must solve the indifference condition:

$$\mathbb{E}p(c) = \mathbb{E}\max\{p(c), u(X_0)\} - \psi$$

which, using and the fact that $\hat{c}(X) = u(X) - \vartheta(X)$ and expanding the expectations, gives:

$$F(\hat{c}(X_0))[\hat{c}(X_0) - \mathbb{E}[c|c \leq \hat{c}(X_0)]] = \psi. \tag{B.2}$$

Observe that the left-hand side of equation $(B.2)$ is a strictly increasing continuous function of $\hat{c}(X_0)$. In particular, the left-hand side of $(B.2)$ equals zero if $\hat{c}(X_0) = \underline{c}$ and it equals $\bar{c} - \mathbb{E}c$ if $\hat{c}(X_0) = \bar{c}$. Therefore, in the region $\psi \in [0, \bar{c} - \mathbb{E}c]$ there exists a unique, continuous, and increasing function of ψ, denoted $c(\psi)$, that takes values in $[\underline{c}, \bar{c}]$ and that solves:

$$F(c(\psi))[c(\psi) - \mathbb{E}[c|c \leq c(\psi)]] = \psi.$$

If $\psi > \bar{c} - \mathbb{E}c$, the equation admits no solution.

To characterize the equilibrium threshold X_0, I distinguish two cases.

Case I. Assume that $\hat{c}(0) \geq \underline{c}$. Define $\underline{\psi}$ as the (unique) value of ψ that satisfies $c(\underline{\psi}) = \hat{c}(0)$. By construction, when search costs are $\underline{\psi}$, type $x = 0$ is indifferent between selling to the intermediary and searching for buyers. Therefore, for all $\psi \leq \underline{\psi}$, type $x = 0$ strictly prefers to search, and, since the payoff from searching is increasing in x, the same will hold for all good asset owners. It follows that the equilibrium threshold X_0 is zero at all $\psi \leq \underline{\psi}$. Since type $x = 0$ is indifferent at $\psi = \underline{\psi}$, the function $X_0(\psi)$ is continuous at $\underline{\psi}$.

Case II. Assume that $\hat{c}(0) < \underline{c}$. The condition implies that $x^*(\underline{c}) > 0$. Clearly, all types $x < x^*(\underline{c})$ anticipate that they would accept every offer at $t = 1$, hence they never search irrespective of ψ. Set $\psi = 0$ and note that $c(0) = \underline{c}$. By construction, when search costs are zero, type $x^*(\underline{c})$ is indifferent between selling to the intermediary and searching for buyers. Therefore, the equilibrium threshold X_0 satisfies $X_0 = x^*(\underline{c})$ at $\psi = 0$. Moreover, the function is continuous at zero.

One can use a similar logic to characterize the threshold X_1. If the equilibrium threshold X_1 is interior, it must solve the indifference condition

$$u(X_1) = \mathbb{E}\max\{p(c), u(X_1)\} - \psi$$

which, using and the fact that $\hat{c}(X) = u(X) - \vartheta(X)$ and expanding the expectations, gives:

$$[1 - F(\hat{c}(X_1))][\mathbb{E}[c|c \geq \hat{c}(X_1)] - \hat{c}(X_1)] = \psi. \tag{B.3}$$

The left-hand side of equation $(B.3)$ is a strictly decreasing continuous function of $\hat{c}(X_1)$. In particular, the left-hand side of $(B.3)$ equals zero if $\hat{c}(X_1) = \bar{c}$ and it equals $\mathbb{E}c - \underline{c}$ if $\hat{c}(X_1) = \underline{c}$. Therefore, in the region $\psi \in [0, \mathbb{E}c - \underline{c}]$, there exists a unique, continuous, and strictly decreasing function of ψ, denoted $C(\psi)$, that solves:

$$[1 - F(C(\psi))][\mathbb{E}[c|c \geq C(\psi)] - C(\psi)] = \psi.$$

If $\psi > \mathbb{E}c - \underline{c}$, the equation admits no solution.

To characterize the equilibrium threshold X_1, I distinguish two cases.

Case I. Assume that $\hat{c}(1) \leq \bar{c}$. Denote $\hat{\psi}$ the (unique) value of search costs such that $C(\hat{\psi}) = \hat{c}(1)$. By construction, when search costs are $\hat{\psi}$, type $x = 1$ is indifferent between searching for buyers and keeping the asset. Therefore, for all $\psi \leq \hat{\psi}$, type $x = 1$ strictly prefers to search, and, since the relative payoff from searching compared to keeping the asset is decreasing in x, the same will hold for all good asset owners. It follows that the equilibrium threshold X_1 equals one at all $\psi \leq \hat{\psi}$. Since type $x = 1$ is indifferent at $\psi = \hat{\psi}$, the function $X_1(\psi)$ is continuous at $\hat{\psi}$.

Case II. Assume that $\hat{c}(1) > \bar{c}$. The condition implies that $x^*(\bar{c}) < 1$. Clearly, all types $x > x^*(\bar{c})$ anticipate that they would reject every offer at $t = 1$, hence they never search irrespective of ψ. Set $\psi = 0$ and note that $C(0) = \bar{c}$. By construction, when search costs are zero, type $x^*(\bar{c})$ is indifferent between searching for buyers and keeping the asset. Therefore, the equilibrium threshold X_1 satisfies $X_1(\psi = 0) = x^*(\bar{c})$. Moreover, the function is continuous at $\psi = 0$.

To summarize, I have identified critical thresholds of ψ such that the indifferent types X_0 and X_1 are, respectively, zero and one when search costs are below such critical thresholds. When search costs are above them, X_0 and X_1 are interior and governed by the indifference conditions $(B.2)$ and $(B.3)$. When interior, X_0 and X_1 can be proven, by differentiating such equations, to be, respectively, strictly increasing and strictly decreasing functions of ψ. Let me denote them $X_0(\psi)$ and $X_1(\psi)$.

In equilibrium, it must be that $X_1(\psi) \geq X_0(\psi)$. Using the indifference conditions $(B.2)$ and $(B.3)$, one can verify that there exists a unique value for search costs, denoted $\overline{\psi}$, such that the function $X_1(\psi)$ crosses the function $X_0(\psi)$ from above. That is, $\overline{\psi}$ solves: $C(\overline{\psi}) = c(\overline{\psi}) = \tilde{c}$. To find $\tilde{c}$, use:

$$[1 - F(\tilde{c})][\mathbb{E}[c|c \geq \tilde{c}] - \tilde{c}] = F(\tilde{c})[\tilde{c} - \mathbb{E}[c|c \leq \tilde{c}]]$$

giving the unique solution:

$$\tilde{c} = \mathbb{E}[c].$$

It follows that $X_0(\overline{\psi}) = X_1(\overline{\psi}) = x^*(\mathbb{E}[c])$.

When search costs exceed $\overline{\psi}$, type $x^*(\mathbb{E}[c])$ is indifferent between selling to the intermediary and keeping the asset, and all types (weakly) below strictly prefer to sell to the intermediary rather than searching for buyers or keeping the asset. Moreover, all types (weakly) above strictly prefer to keep the asset rather than searching for buyers or selling to the intermediary. Hence, $X_0(\psi) = X_1(\psi) = x^*(\mathbb{E}[c])$ for all $\psi \geq \overline{\psi}$.

Clearly, $\underline{c} < \mathbb{E}[c] < \bar{c}$ and therefore $\overline{\psi} < \min\{\mathbb{E}c - \underline{c}, \bar{c} - \mathbb{E}c\}$. Hence, whenever some sellers search in equilibrium (that is, $X_1(\psi) \geq X_0(\psi)$), ψ lies in the domain in which the solutions $C(\psi)$ and $c(\psi)$ are unique and strictly monotone.

To show that both $X_0(\psi)$ and $X_1(\psi)$ are also continuous in the regions in which they are interior, one can use the same logic behind the proof of Proposition 1. That is, for the function $X_0(\psi)$, rewrite the indifference condition as:

$$F\left(u\left(X_0\right)-\vartheta\left(X_0\right)\right)\left[u\left(X_0\right)-\vartheta\left(X_0\right)-\mathbb{E}\left[c \mid c \leq u\left(X_0\right)-\vartheta\left(X_0\right)\right]\right]-\psi=0$$

and use the fact that the left-hand side of the condition is increasing in X_0 (because $u'(X)>\vartheta'(X)$ for all X due to A4) and decreasing in ψ. A symmetric logic applies to the function $X_1(\psi)$. This terminates the proof. ■

Proof of Theorem 2. In the region $\psi \leq \min\left\{\underline{\psi}, \hat{\psi}\right\}$, it holds that $\min\left\{\underline{c}, u\left(X_0(\psi)\right)-\vartheta\left(X_0(\psi)\right)\right\}=\underline{c}$, and that $\max\left\{\bar{c}, u\left(X_1(\psi)\right)-\vartheta\left(X_1(\psi)\right)\right\}=\bar{c}$, and $X_0(\psi)=x^*\left(\underline{c}\right)$ and $X_1(\psi)=x^*(\bar{c})$. Welfare is then:

$$W(\psi)=\lambda\left[\int_{\underline{c}}^{\bar{c}} x^*(c)(v+c)+\int_{x^*(c)}^1 u(x)\,dx\right] dF(c)$$
$$+(1-\lambda)\mathbb{E}c-\lambda\left[x^*(\bar{c})-x^*\left(\underline{c}\right)\right]\psi$$

which is strictly decreasing in ψ. Distinguish now two cases.

Case I. If $\min\left\{\underline{\psi}, \hat{\psi}\right\}=\hat{\psi}$, then, in the region $\hat{\psi} \leq \psi \leq \underline{\psi}$, it holds that $\min\left\{\underline{c}, u\left(X_0(\psi)\right)-\vartheta\left(X_0(\psi)\right)\right\}=\underline{c}$, and $X_0(\psi)=x^*\left(\underline{c}\right)$.

Moreover, $\max\left\{\bar{c}, u\left(X_1(\psi)\right)-\vartheta\left(X_1(\psi)\right)\right\}=u\left(X_1(\psi)\right)-\vartheta\left(X_1(\psi)\right)$ with $X_1(\psi)$ being interior and disciplined by the indifference condition (B.3). Welfare is then:

$$\begin{aligned} W(\psi) &= +\lambda \int_{\underline{c}}^{u(X_1(\psi))-\vartheta(X_1(\psi))}\left[x^*(c)(v+c)+\int_{x^*(c)}^1 u(x)\,dx\right] dF(c) \\ &+ \lambda \int_{u(X_1(\psi))-\vartheta(X_1(\psi))}^{\bar{c}}\left[X_1(\psi)(v+c)+\int_{X_1(\psi)}^1 u(x)\,dx\right] dF(c) \\ &+ (1-\lambda)\mathbb{E}c \\ &- \lambda\left[X_1(\psi)-x^*\left(\underline{c}\right)\right]\psi. \end{aligned}$$

Differentiating it with respect to ψ gives:

$$\frac{dW(\psi)}{d\psi}=\lambda\left\{\int_{u(X_1(\psi))-\vartheta(X_1(\psi))}^{\bar{c}} \frac{dX_1(\psi)}{d\psi}\left[v+c-u\left(X_1(\psi)\right)\right] dF(c)\right.$$
$$\left.-X_1(\psi)+x^*\left(\underline{c}\right)-\psi \frac{dX_1(\psi)}{d\psi}\right\}.$$

Substituting ψ with equation (B.3) gives:

$$\frac{dW(\psi)}{d\psi}=\lambda\left\{\frac{dX_1(\psi)}{d\psi}\left[1-F\left(u\left(X_1(\psi)\right)-\vartheta\left(X_1(\psi)\right)\right)\right]\left[v-\vartheta\left(X_1(\psi)\right)\right]\right.$$
$$\left.-X_1(\psi)+x^*\left(\underline{c}\right)\right\}.$$

Using the Implicit Function Theorem,

$$\frac{dX_1(\psi)}{d\psi}=-\left\{\left[\left(1-F\left(u\left(X_1(\psi)\right)-\vartheta\left(X_1(\psi)\right)\right)\right)\right]\left[u'\left(X_1(\psi)\right)-\vartheta'\left(X_1(\psi)\right)\right]\right\}^{-1}<0$$

and therefore, $\frac{dW(\psi)}{d\psi}<0$.

In the region $\underline{\psi} \leq \psi \leq \bar{\psi}$, it holds that $\min\left\{\underline{c}, u\left(X_0(\psi)\right)-\vartheta\left(X_0(\psi)\right)\right\}=u\left(X_0(\psi)\right)-\vartheta\left(X_0(\psi)\right)$.

Moreover, $\max\left\{\bar{c}, u\left(X_1(\psi)\right)-\vartheta\left(X_1(\psi)\right)\right\}=u\left(X_1(\psi)\right)-\vartheta\left(X_1(\psi)\right)$, with both $X_0(\psi)$ and $X_1(\psi)$ being interior and disciplined by the indifference conditions (B.2) and (B.3). Using similar steps as above, one gets that:

$$\begin{aligned} \frac{dW(\psi)}{d\psi} = & \lambda\left\{\frac{dX_0(\psi)}{d\psi}\left[F\left(u\left(X_0(\psi)\right)-\vartheta\left(X_0(\psi)\right)\right)\right]\left[v-\vartheta\left(X_0(\psi)\right)\right]+X_0(\psi)\right\} \\ & +\lambda\left\{\frac{dX_1(\psi)}{d\psi}\left[1-F\left(u\left(X_1(\psi)\right)-\vartheta\left(X_1(\psi)\right)\right)\right]\left[v-\vartheta\left(X_1(\psi)\right)\right]-X_1(\psi)\right\}. \end{aligned}$$

To sign $\frac{dW(\psi)}{d\psi}$, use the fact that

$$\frac{dX_0(\psi)}{d\psi}=+\left\{\left[F\left(u\left(X_0(\psi)\right)-\vartheta\left(X_0(\psi)\right)\right)\right]\left[u'\left(X_0(\psi)\right)-\vartheta'\left(X_0(\psi)\right)\right]\right\}^{-1}>0$$

and observe that the function:

$$[v-\vartheta(x)]\left[u'(x)-\vartheta'(x)\right]^{-1}+x$$

is strictly decreasing everywhere if condition (⋆) holds. Therefore, if (⋆) holds, welfare is strictly increasing in search costs, that is $\frac{dW(\psi)}{d\psi}>0$.

The last region to consider is $\psi \geq \bar{\psi}$, in which $X_0(\psi)=X_1(\psi)=x^*(\mathbb{E}c)$. Since $X_0(\psi)$ and $X_1(\psi)$ are constant and no agent searches in equilibrium, $\frac{dW(\psi)}{d\psi}=0$.

Case II. Assume now that $\min\left\{\underline{\psi}, \hat{\psi}\right\}=\underline{\psi}$. In the region $\underline{\psi} \leq \psi \leq \hat{\psi}$, it holds that $\max\left\{\bar{c}, u\left(X_1(\psi)\right)-\vartheta\left(X_1(\psi)\right)\right\}=\bar{c}$, and $X_1(\psi)=x^*(\bar{c})$.

Moreover, $\min\left\{\underline{c}, u\left(X_0(\psi)\right)-\vartheta\left(X_0(\psi)\right)\right\}=u\left(X_0(\psi)\right)-\vartheta\left(X_0(\psi)\right)$ and $X_0(\psi)$ is interior and disciplined by (B.2). Welfare is then:

$$\begin{aligned} W(\psi) &= +\lambda \int_{\underline{c}}^{u(X_0(\psi))-\vartheta(X_0(\psi))}\left[X_0(\psi)(v+c)+\int_{X_0(\psi)}^1 u(x)\,dx\right] dF(c) \\ &+ \lambda \int_{u(X_0(\psi))-\vartheta(X_0(\psi))}^{\bar{c}}\left[x^*(c)(v+c)+\int_{x^*(c)}^1 u(x)\,dx\right] dF(c) \\ &+ (1-\lambda)\mathbb{E}c \\ &- \lambda\left[x^*(\bar{c})-X_0(\psi)\right]\psi. \end{aligned}$$

Differentiating it with respect to ψ gives:

$$\frac{dW(\psi)}{d\psi}=\lambda\left\{\int_{\underline{c}}^{u(X_0(\psi))-\vartheta(X_0(\psi))} \frac{dX_0(\psi)}{d\psi}\left[v+c-u\left(X_0(\psi)\right)\right] dF(c)\right.$$
$$\left.-x^*(\bar{c})+X_0(\psi)+\psi \frac{dX_0(\psi)}{d\psi}\right\}.$$

Substitute the expression for $\frac{dX_0(\psi)}{d\psi}$ derived above, to get:

$$\frac{dW(\psi)}{d\psi}=\left[v-\vartheta\left(X_0(\psi)\right)\right]\left[u'\left(X_0(\psi)\right)-\vartheta'\left(X_0(\psi)\right)\right]^{-1}+X_0(\psi)-x^*(\bar{c}).$$

Since the function

$$[v-\vartheta(x)]\left[u'(x)-\vartheta'(x)\right]^{-1}+x$$

is strictly decreasing everywhere under condition (⋆), and since $v>\vartheta(x)$ and $u'(x)>\vartheta'(x)$ for all x, it follows that $\frac{dW(\psi)}{d\psi}>0$. This terminates the proof. ■

Proof of Propositions 7 and 8. First, I characterize the equilibrium in an economy in which intermediaries do not exist for all possible values of ψ. Sellers are left with the choice whether to pay the cost ψ search for buyers at $t=1$, or keep their asset until the end of period $t=1$. Given a market price $p(c)$ prevailing in every state, the expected payoff from searching is:

$$\mathbb{E}\max\{p(c), u(x)\}-\psi$$

while the payoff from retaining the asset is $u(x)$. Since the payoff from retaining increases in x more steeply then the payoff from searching, the strategy profile of good asset owners can be described by a cutoff, denoted $\tilde{X}$, such that sellers of good assets search if and only if $x \leq \tilde{X}$. If some sellers of good assets find it optimal to search, then sellers of bad assets, whose payoff from retaining is zero, will also search. To find $\tilde{X}$ in such an interior equilibrium, use the indifference condition:

$$u\left(\tilde{X}\right)=\mathbb{E}\max\left\{p(c), u\left(\tilde{X}\right)\right\}-\psi$$

which, using and the definition $\hat{c}(X)=u(X)-\vartheta(X)$ and expanding the expectations, gives:

$$\left[1-F\left(\hat{c}\left(\tilde{X}\right)\right)\right]\left[\mathbb{E}\left[c \mid c \geq \hat{c}\left(\tilde{X}\right)\right]-\hat{c}\left(\tilde{X}\right)\right]=\psi.$$

Observe that the equation above is the same as (B.3), and therefore, $\tilde{X}(\psi)=X_1(\psi)$. Recall that the function $X_1(\psi)$ is strictly decreasing, and define the unique $\tilde{\psi}$ that solves $X_1(\tilde{\psi})=0$. Note that $\tilde{\psi} \geq \overline{\psi}$, since $X_1\left(\overline{\psi}\right)=x^*(\mathbb{E}c) \geq 0$. Indeed, $\tilde{\psi}$ solves:

$$u(0)=\mathbb{E}\max\{p(c), u(0)\}-\tilde{\psi}$$

with $p(c)=\vartheta(0)+c$ when $c \geq c_1$ and $p(c)=c$ when $c<c_1$. Since $\vartheta(0)=0$, we have:

$$u(0)=\mathbb{E}\max\{c, u(0)\}-\tilde{\psi}$$

For $\psi>\tilde{\psi}$, no good asset owner searches, and therefore only bad assets are traded and are priced fairly, that is, $p(c)=c$, as long as bad asset owners find it optimal to search:

$$\mathbb{E}p(c) - \psi = \mathbb{E}c - \psi \geq 0.$$

Clearly, $\tilde{\psi} \leq \mathbb{E}c$, and the inequality is strict as long as $u(0) > 0$.

For search costs $\psi \geq \mathbb{E}c$, no seller searches in equilibrium.

If $\psi \leq \mathbb{E}c$, expected welfare in the market without intermediaries is a function of ψ denoted $W_N(\psi)$, given by:

$$\begin{array}{rcl} W_N(\psi) & = & \lambda \int_{\underline{c}}^{\max\{\bar{c}, u(X_1(\psi)) - \vartheta(X_1(\psi))\}} \left[x^*(c)(v+c) + \int_{x^*(c)}^1 u(x)\,dx\right] dF(c) \\ & + & \lambda \int_{\max\{\bar{c}, u(X_1(\psi)) - \vartheta(X_1(\psi))\}}^{\bar{c}} \left[X_1(\psi)(v+c) + \int_{X_1(\psi)}^1 u(x)\,dx\right] dF(c) \\ & + & (1-\lambda)\mathbb{E}c \\ & - & \left[\lambda X_1(\psi) + 1 - \lambda\right]\psi. \end{array}$$

If $\psi > \mathbb{E}c$, instead:

$$W_N(\psi) = \lambda \int_0^1 u(x)\,dx.$$

Since $X_1(\psi)$ is continuous, $W_N(\psi)$ is continuous for all $\psi \leq \mathbb{E}c$. Moreover, one can verify that $W_N(\psi)$ is also continuous at $\psi = \mathbb{E}c$. Since $W_N(\psi)$ is constant for $\psi > \mathbb{E}c$, it follows that $W_N(\psi)$ is continuous for every $\psi \geq 0$.

To compare expected welfare with and without intermediaries, observe that, when intermediaries are present, expected welfare for a given ψ is:

$$\begin{array}{rcl} W(\psi) & = & \lambda \int_{\underline{c}}^{\min\{\underline{c}, u(X_0(\psi)) - \vartheta(X_0(\psi))\}} \left[X_0(\psi)(v+c) + \int_{X_0(\psi)}^1 u(x)\,dx\right] dF(c) \\ & + & \lambda \int_{\min\{\underline{c}, u(X_0(\psi)) - \vartheta(X_0(\psi))\}}^{\max\{\bar{c}, u(X_1(\psi)) - \vartheta(X_1(\psi))\}} \left[x^*(c)(v+c) + \int_{x^*(c)}^1 u(x)\,dx\right] dF(c) \\ & + & \lambda \int_{\max\{\bar{c}, u(X_1(\psi)) - \vartheta(X_1(\psi))\}}^{\bar{c}} \left[X_1(\psi)(v+c) + \int_{X_1(\psi)}^1 u(x)\,dx\right] dF(c) \\ & + & (1-\lambda)\mathbb{E}c \\ & - & \lambda\left[X_1(\psi) - X_0(\psi)\right]\psi. \end{array}$$

Consider now the case in which $\psi < \mathbb{E}c$. The difference in welfare under the two scenarios can then be expressed as:

$$\begin{array}{rcl} W(\psi) - W_N(\psi) & = & \lambda \int_{\underline{c}}^{\min\{\underline{c}, u(X_0(\psi)) - \vartheta(X_0(\psi))\}} \int_{x^*(c)}^{X_0(\psi)} [v + c - u(x)\,dx]\, dF(c) \\ & + & \left[\lambda X_0(\psi) + 1 - \lambda\right]\psi. \end{array}$$

In the region in which $\psi \leq \underline{\psi}$, it holds that $\min\left\{\underline{c}, u\left(X_0(\psi)\right) - \vartheta\left(X_0(\psi)\right)\right\} = \underline{c}$, thus $W(\psi) > W_N(\psi)$.

If instead $\overline{\psi} > \psi > \underline{\psi}$, evaluate the difference $W(\psi) - W_N(\psi)$ by substituting the term $X_0(\psi)\psi$ from equation (12), to get:

$$\begin{array}{rcl} W(\psi) - W_N(\psi) & = & \lambda \int_{\underline{c}}^{u(X_0(\psi)) - \vartheta(X_0(\psi))} \int_{x^*(c)}^{X_0(\psi)} \left[v - \vartheta\left(X_0(\psi)\right) + u\left(X_0(\psi)\right) - u(x)\,dx\right] dF(c) \\ & + & \lambda \int_{\underline{c}}^{u(X_0(\psi)) - \vartheta(X_0(\psi))} x^*(c)\left[u\left(X_0(\psi)\right) - \vartheta\left(X_0(\psi)\right) - c\,dx\right] dF(c) \\ & + & (1-\lambda)\psi \end{array}$$

which is strictly positive.

The last region to consider is $\psi \geq \overline{\psi}$. Note that $W\left(\overline{\psi}\right) > W_N\left(\overline{\psi}\right)$ and that $W(\psi)$ is constant for all $\psi \geq \bar{\psi}$. Thus, since I have established that $W_N(\psi)$ is continuous, it will suffice to show that $W_N(\psi)$ is decreasing in the region $\psi \geq \tilde{\psi}$. When $X_1(\psi)$ is interior, by direct calculation:

$$\frac{dW_N(\psi)}{d\psi} = \lambda \frac{dX_1(\psi)}{d\psi} \lambda \int_{u(X_1(\psi)) - \vartheta(X_1(\psi))}^{\bar{c}} v - \vartheta\left(X_1(\psi)\right) dF(c) - \lambda X_1(\psi) - 1 + \lambda$$

which is negative because $X_1(\psi)$ is decreasing.

When $\mathbb{E}c \geq \psi \geq \tilde{\psi}$, so that $X_1(\psi) = 0$ while bad asset holders search, it holds that $\frac{dW_N(\psi)}{d\psi} = -(1-\lambda)$. Finally, if $\psi > \mathbb{E}c$, $\frac{dW_N(\psi)}{d\psi} = 0$. ■

Proof of Proposition 9. For expositional reasons, it is useful to define the function:

$$\varphi(t) := \mathbb{E}[c | c \geq t].$$

In words, the function $\varphi(t)$ is the expectation of the shock conditionally on being above any threshold $t \in [\underline{c}, \bar{c}]$.

Step 1. First, I prove that the marginal good asset owner who is indifferent between accepting the contract and retaining the asset until $t = 1$ is the seller who is indifferent whether to sell or retain in the direct market when the shock equals the expectation of c conditional on c belonging to the set where the intermediary commits to sell under contract $\boldsymbol{\sigma}$, that is, conditional on $c \in \{c : \sigma(c) = 1\}$.

Clearly, all sellers with bad assets would enter the contract. By monotonicity of $u(x)$ and observing that, when the intermediary sells, all sellers receive the same price, it must be that sellers enter the contract if and only if their type x is below a threshold. Denote such threshold $\tilde{x}$. Observe that it is without loss of generality to restrict attention to contracts in which, in those states in which the seller is given discretion, the indifferent type $\tilde{x}$ will reject the offer. The reason is that in those states in which the good asset owner with the highest valuation prefers to accept the offer, every seller will accept it too, therefore the choice whether to give sellers discretion or not is immaterial and cannot affect total surplus. Formally, for all c such that $\sigma(c) = 0$, impose $p^b \leq u(\tilde{x})$. One can then characterize $\tilde{x}$ by using the indifference condition:

$$\int \sigma(c)\, p^b(c; \boldsymbol{\sigma}) + [1 - \sigma(c)]\, u(\tilde{x})\, dF(c) = u(\tilde{x})$$

where

$$p^b(c; \boldsymbol{\sigma}) = \frac{\lambda[\sigma(c)\tilde{x} + (1 - \sigma(c))\, x(c; \boldsymbol{\sigma})]}{\lambda[\sigma(c)\tilde{x} + (1 - \sigma(c))\, x(c; \boldsymbol{\sigma})] + 1 - \lambda} v + c$$

and $x(c; \boldsymbol{\sigma})$ is the highest solution to

$$p^b(c; \boldsymbol{\sigma}) = u(x(c; \boldsymbol{\sigma})).$$

Divide both sides of the indifference condition characterizing $\tilde{x}$ by $\int \sigma(c)\, dF(c)$ to get:

$$\frac{\int \sigma(c)\, p^b(c; \boldsymbol{\sigma})\, dF(c)}{\int \sigma(c)\, dF(c)} = u(\tilde{x})$$

and thus:

$$\frac{\int_{c:\,\sigma(c)=1} \left(\frac{\lambda\tilde{x}}{\lambda\tilde{x} + 1 - \lambda} v\right) dF(c)}{\int_{c:\,\sigma(c)=1} dF(c)} + \mathbb{E}[c | c : \sigma(c) = 1] = u(\tilde{x})$$

which proves that $\tilde{x} = x^*(\hat{c})$, where $\hat{c} := \mathbb{E}[c | c : \sigma(c) = 1]$.

Step 2. Here I derive the marginal benefits and costs of setting $\sigma = 0$ versus $\sigma = 1$ at any given c, for a given contract $\boldsymbol{\sigma}$. Note that whenever $\sigma(c) = 0$, the solution $x(c; \boldsymbol{\sigma})$ is given by:

$$\frac{\lambda x(c; \boldsymbol{\sigma})}{\lambda x(c; \boldsymbol{\sigma}) + 1 - \lambda} v + c = u(x(c; \boldsymbol{\sigma}))$$

thus, $x(c; \boldsymbol{\sigma}) = x^*(c)$. The marginal cost of giving discretion to sellers in state c is that types in $[x^*(c), x^*(\hat{c})]$ would no longer sell the asset, causing a surplus loss:

$$f(c) \int_{x^*(c)}^{x^*(\hat{c})} [v + c - u(x)]\, dx.$$

The marginal benefit is instead given by the effect on surplus of the change $x^*(\hat{c})$ induced by the change in $\hat{c}$:

$$\frac{f(c)}{\int_{c:\,\sigma(c)=1} dF(c)} (\hat{c} - c)\, x^{*'}(\hat{c}) \int_{c:\,\sigma(c)=1} v + c - u(x(\hat{c}))\, dF(c)$$

which can be rewritten as:

$$f(c)(\hat{c} - c)\, x^{*'}(\hat{c})\, [v + \hat{c} - u(x(\hat{c}))].$$

Step 3. I now show that, in every optimal contract, $\sigma(c) = 1$ for all $c \geq c_1$. Thus, whenever sellers have discretion, every seller with the good asset keeps it, that is $x^*(c) = 0$ for all c such that $\sigma(c) = 0$. To prove it, assume, in anticipation of a contradiction, that there are states $c > c_1$ such that, in the optimal contract, $\sigma(c) = 0$. Observe that the expected surplus generated by the contract is the same as if the market was direct, but the distribution of c was replaced by $\tilde{F}$, whose density mimics f in the states where $\sigma(c) = 0$, is zero in the states c such that $\sigma(c) = 1$, and admits a mass point at $\hat{c}$ associated to total probability mass $\int_{\{c:\,\sigma(c)=1\}} dF(c)$. Note that it must be the case that $\hat{c} > c$ for all states such that $\sigma(c) = 0$ otherwise a deviation at c would only have a positive marginal benefit. Thus, $\hat{c} > c_1$. Consider now a deviation to a contract whereby, for all $c \geq c_1$, $\sigma(c) = 1$. The expected surplus generated by the contract is the same as if the market was direct, but the distribution of c was replaced by $\hat{F}$. For values $c < c_1$, $\tilde{F}$ and $\hat{F}$ coincide, and it is easy to see that, for values $c \geq c_1$, $\tilde{F}$ is a mean-preserving spread of $\hat{F}$. Since condition $(\star)$ holds, surplus in the direct market is a strictly concave function of c for $c \geq c_1$, making the deviation profitable.

Step 4. In all states $c \leq c_1$, the marginal cost of giving seller discretion can be written as:

$$f(c) \int_0^{x^*(\hat{c})} [v + c - u(x)]\, dx$$

where I have used the fact that, since all states c where $\sigma(c) = 0$ are below c_1, it holds that $x^*(c) = 0$. In Step 3 I have proven that, if ever, the optimal policy gives sellers discretion only in states below c_1. Observe now that if the marginal benefit exceeds the marginal cost at some c, the same would hold for all values of the shock below c. Thus the optimal contract must take a partitional form: $\sigma(c) = 0$ if $c \leq c'$ and $\sigma(c) = 1$ otherwise. Thus, it holds that $\hat{c} = \varphi(c') = \mathbb{E}[c | c \geq c']$. The optimal threshold c' is the solution to:

$$\int_0^{x^*(\varphi(c'))} v + c' - u(x)\, dx = (\varphi(c') - c')\, x^{*'}(\varphi(c'))\, [v + \varphi(c') - u(x^*(\varphi(c')))] \tag{B.4}$$

whenever such an interior solution exists.

I now characterize the optimal threshold c'. I distinguish two cases.

Case I. Assume $\mathbb{E}c > c_1 > \underline{c}$. At a candidate solution $c' = \underline{c}$, it holds that $\varphi(c') = \mathbb{E}c$. Here, the marginal cost of giving discretion to sellers exceeds its marginal benefit if and only if

$$\int_0^{x^*(\mathbb{E}c)} v + \underline{c} - u(x)\, dx \geq (\mathbb{E}c - \underline{c}) \left[u'(x^*(\mathbb{E}c)) - \vartheta'(x^*(\mathbb{E}c))\right]^{-1} [v + \mathbb{E}c - u(x^*(\mathbb{E}c))].$$

Clearly if the condition above holds, then $\sigma(c) = 1$ for all c in the optimal contract. Note that the condition is implied by $(\star\star)$. Also, $(\star\star)$ can only hold when $\mathbb{E}c > c_1 > \underline{c}$, proving that under $(\star\star)$ it is optimal to fully delegate to the intermediary.

If the condition does not hold, then in the optimal contract $\sigma(\underline{c}) = 0$. Optimality is found if marginal costs and benefit equate at some c'.

I prove now that a unique such interior solution must exist and, in particular, such a solution lies in the open interval: $(\underline{c}, c_1)$.

To do so, for every c, define the variable $\tilde{c}$ implicitly as:

$$\int_0^{x^*(\tilde{c})} v + \tilde{c} - u(x)\, dx = (\tilde{c} - c) \left\{ x^{*'}(\tilde{c}) [v + \tilde{c} - u(x^*(\tilde{c}))] + x^*(\tilde{c}) \right\}$$

Denote, respectively, $\mu^1(\tilde{c}, c)$ and $\mu^2(\tilde{c}, c)$ the left-hand side and the right-hand side of the equation above. The fact that condition $(\star)$ holds implies that

$$x^{*'}(c) [v + c - u(x^*(c))] + x^*(c)$$

is strictly decreasing in c. In turn, this means $\mu^1_{\tilde{c}}(\tilde{c}, c) > \mu^2_{\tilde{c}}(\tilde{c}, c)$. By totally differentiating the equation defining $\tilde{c}(c)$, it then follows that $\tilde{c}(c)$ is strictly decreasing. Moreover, the case I am considering implies that $\mu^1(\mathbb{E}c, \underline{c}) < \mu^2(\mathbb{E}c, \underline{c})$, from which it follows that $\tilde{c}(\underline{c}) > \mathbb{E}c$. Define $c^h := \tilde{c}(\underline{c})$. I now derive the inequality: $\tilde{c}^{-1}(\mathbb{E}c) < c_1$. To prove it, it suffices to observe that $\mu^1(\mathbb{E}c, c_1) > \mu^2(\mathbb{E}c, c_1)$. This holds since:

$$\begin{aligned}
\mu^2(\mathbb{E}c, c_1) &= (\mathbb{E}c - c_1)\{x^{*'}(\mathbb{E}c)[v + \mathbb{E}c - u(x^*(\mathbb{E}c))] + x^*(\mathbb{E}c)\} \\
&< \int_0^{x^*(\mathbb{E}c)} (v + \mathbb{E}c)\, dx + \int_{x^*(\mathbb{E}c)}^1 u(x)\, dx - \int_0^{x^*(c_1)} (v + c_1)\, dx - \int_{x^*(c_1)}^1 u(x)\, d \\
&< \int_0^{x^*(\mathbb{E}c)} v + \mathbb{E}c - u(x)\, dx \\
&= \mu^1(\mathbb{E}c, c_1)
\end{aligned}$$

where the first inequality is due to condition $(\star)$, implying concavity of the function:

$$\int_0^{x^*(c)} (v + c)\, dx + \int_{x^*(c)}^1 u(x)\, dx$$

and the second inequality is derived by adding and subtracting to the left-hand side of the inequality the term $\int_0^{x^*(c_1)} u(x)\, dx$ and noting that $v + c_1 > u(x^*(c_1))$. I have established that $\tilde{c}(c)$ exists, and is a unique, strictly decreasing function taking values in $[\mathbb{E}c, c^h]$ in the domain $[\underline{c}, \tilde{c}^{-1}(\mathbb{E}c) < c_1]$. An optimal cutoff c' must satisfy:

$$\tilde{c}(c') = \varphi(c').$$

For every distribution $F(c)$ the function $\varphi(c') = \mathbb{E}[c | c \geq c']$ is weakly increasing in c'. Noting that $\varphi(\underline{c}) = \mathbb{E}c < \varphi(\tilde{c}^{-1}(\mathbb{E}c))$, it follows that a unique $c' \in (\underline{c}, c_1)$ exists.

Case II. Assume $c_1 > \mathbb{E}[c] > \underline{c}$. In this case, there exists a value of c, denoted c'', such that $\varphi(c'') = c_1$. For all values of c' below c'', one can verify $x^*(\hat{c}) = x^*(c') = 0$. Thus, marginal benefit and marginal cost are both zero. At $c' = c''$, they jump discontinuously. If the marginal cost of giving discretion to sellers at c'' exceeds its benefits, then it is optimal to set $c' = c''$. If not, then the arguments of Case 1 can be repeated to prove the existence of an optimal solution c' that lies in (c'', c_1). ■

Proof of Corollaries 1 and 2. The proof of Corollary 1 is straightforward.

To prove Corollary 2, observe first that, if the optimal contract coincides with the full delegation contract, $\triangle(x, \theta)$ is trivially zero for all (x, θ). Moreover, observe that under the baseline full delegation contract, sellers with bad assets and those with good assets and type below of $x^*(\mathbb{E}c)$ obtain the same expected payoff:

$$\mathbb{E}p^b(c) = \mathbb{E}[\vartheta(x^*(\mathbb{E}c)) + c] = \vartheta(x^*(\mathbb{E}c)) + \mathbb{E}c$$

whereas all other sellers retain and obtain $u(x)$ in every state. Denote with σ^* the optimal contract and consider now the case in which it does not coincide with the full delegation contract, that is $c' > \underline{c}$. Compute the expected payoff to a seller with a good asset and type below $x^*(\hat{c})$ as:

$$F(c')\, u(x) + \int_{c \geq c'} p^b(c, \sigma^*)\, dF = F(c')\, u(x) + \vartheta(x^*(\hat{c})) + \hat{c}$$

where I have used the fact that the optimal contract features $c' < c_1$, hence all sellers with good assets reject the offer when they have discretion. For sellers with bad assets, the expected payoff under the optimal contract is instead:

$$\mathbb{E}p^b(c, \sigma^*) = \int_{c < c'} c\, dF + \vartheta(x^*(\hat{c})) + \hat{c}$$

where I have used the fact that, since all sellers with good assets reject in states below c', competitive bidding implied that $p^b(c, \sigma^*) = c$ for all $c < c'$. Computing $\triangle(x, \theta)$ for all types directly leads to the results. To verify that cases exist in which some sellers are better off under the full delegation contract, I construct an example where this is the case. Assume that sellers' valuations are distributed uniformly, with $u \sim U\left[\frac{1}{3}, 2\right]$, so that $u(x) = \frac{5}{3}x + \frac{1}{3}$. Set $v = 3$ and $\lambda = \frac{1}{3}$ and verify that condition $(\star)$ holds. Assume that c is distributed uniformly, with $c \sim U\left[0, \frac{19}{10}\right]$, so that $\mathbb{E}c = \frac{19}{20}$. Note the example is constructed in such a way that, in the baseline full delegation contract, participation is relatively large. Indeed, verify from the indifference condition that the highest solution to (2) at $c = \mathbb{E}c$ is given by $x^*(\mathbb{E}c) \simeq 0.95$. The payoff to any seller accepting the full delegation contract is given by:

$$\mathbb{E}p^b = \vartheta(x^*(\mathbb{E}c)) + \mathbb{E}c \simeq 1.92.$$

By Proposition 9, the surplus-maximizing contract is characterized by a threshold c' that, if interior, solves the optimality condition $(B.4)$. Using the uniform distribution assumption, observe that $\varphi(c') := \mathbb{E}[c | c \geq c'] = \frac{c'}{2} + \frac{19}{20}$. Verify that the optimality condition is satisfied at $c' = 0.06$, so the optimal contract gives sellers discretion for $c \leq c'$, where all good asset holders choose to retain. To find the marginal seller participating in the contract, verify that the highest solution to (2) at $c = \frac{0.06}{2} + \frac{19}{20}$ is given by $x^*(\varphi(c') = \hat{c}) \simeq 0.98$. To verify that some sellers with good assets and sufficiently low valuation are worse off under the optimal contract, write the expected payoff to a seller with a good asset and type $x = 0$ as:

$$F(0.06)\,u(0) + \int_{c\geq 0.06} p^b\left(c,\sigma^*\right) dF = F(0.06)\,u(0) + \vartheta\left(x^*(\hat{c})\right) + \hat{c}.$$

Using the fact that F is uniform in $\left[0, \frac{19}{10}\right]$, that $u(0) = \frac{1}{3}$ and

$$\vartheta\left(x^*(\hat{c})\right) + \hat{c} = u\left(x^*\left(\varphi\left(c'\right) = \hat{c}\right)\right) \simeq u(0.98) = \frac{5}{3}0.98 + \frac{1}{3},$$

verify that the payoff to type $x = 0$ under the optimal contract is below the payoff under the baseline. ■

Proof of Corollary 3. The statement of Corollary 3 pertains to the cases where the optimal threshold c' is interior. Thus, the optimality condition derived in the proof of Proposition 9 has to hold. The right-hand side of such a condition is always strictly positive, implying that:

$$\int_0^{x^*(\hat{c})} v + c' - u(x)\,dx > 0.$$

Hence, it must be that, at c', and for some for $x \leq x^*(\hat{c})$, there are gains from trade: $v + c' > u(x)$. By continuity, this must also be true for some states $c \in \left(\underline{c}, c'\right)$. ■

Proof of Proposition 10. If

$$u^i \leq \frac{x^*(\mathbb{E}c)\,\lambda}{x^*(\mathbb{E}c)\,\lambda + 1 - \lambda} v + \underline{c}$$

holds, intermediaries bid at $t = 0$ is the same as what it would in the model where $u^i = 0$, hence the measure of sellers with good assets accepting the offer is $X^* = x^*(\mathbb{E}c)$, and is not a function of u^i (in the Proof of Proposition 11, the general function $X^*\left(u^i\right)$ is characterized and it is indeed flat and equals $x^*(\mathbb{E}c)$ in the region where u^i is low). Since strategies and the equilibrium allocation are the same as in the baseline model, all results of Sections 3, 4, and 5 apply. ■

Proof of Proposition 11. The equilibrium in the intermediated market is characterized by two cutoffs: X identifies the marginal seller of the good asset at $t = 0$, and c^i represents the lowest realization of c such that intermediaries with good assets choose to sell to final buyers at $t = 1$. The two cutoffs solve simultaneously:

$$\begin{cases} \mathbb{E}\max\left\{\vartheta(X) + c, u^i\right\} = u(X) \\ u^i = \vartheta(X) + c^i \end{cases}$$

leading to the equilibrium condition

$$F\left(u^i - \vartheta(X)\right) u^i + \int_{u^i - \vartheta(X)}^{\bar{c}} \vartheta(X) + c\,dF(c) = u(X). \tag{B.5}$$

An interior equilibrium X is the highest solution to $(B.5)$. Using arguments analogous to those used to characterize the function $x^*(c)$, it is possible to prove that, if $u'(x) > \vartheta'(X)$ everywhere, the equilibrium X is a continuous function of u^i described by:

$$X\left(u^i\right) = \begin{cases} x^*(\mathbb{E}c) & \text{if} \quad u^i < u_1 \\ \sup\left\{X \in [0,1] : \mathbb{E}\max\left\{\vartheta(X) + c, u^i\right\} = u(X)\right\} & \text{if} \quad u_1 \leq u^i < u_2 \\ 1 & \text{if} \quad u_2 \leq u^i. \end{cases}$$

The threshold u_1 solves $u_1 = \vartheta\left(x^*(\mathbb{E}c)\right) + \underline{c}$ and the threshold u_2 solves $\mathbb{E}\max\left\{\vartheta(1) + c, u_2\right\} = u(1)$.

Total expected surplus in the intermediated market is given by:

$$\begin{aligned} \mathbb{E}S_I \quad = \quad & (1-\lambda)\,c + \lambda \int_{\underline{c}}^{\bar{c}} \int_{X(u^i)}^1 u(x)\,dx\,dF(c) \\ + \quad & \lambda X\left(u^i\right)\left[\int_{u^i - \vartheta(X(u^i))}^{\bar{c}} (v + c)\,dF(c) + \int_{\underline{c}}^{u^i - \vartheta(X(u^i))} u^i\,dF(c)\right]. \end{aligned}$$

In Theorem 1, I have derived conditions such that $\mathbb{E}S_I\left(u^i\right) > \mathbb{E}S_D$ for all $u^i < u_1$. Since $X\left(u^i\right)$ is continuous, observe that $\mathbb{E}S_I$ is continuous in u^i. Hence, a sufficient condition to have $\mathbb{E}S_I\left(u^i\right) > \mathbb{E}S_D$ is that $\mathbb{E}S_I\left(u^i\right)$ is increasing in u^i. In Appendix A.1, I provide the expression for the derivative of $\mathbb{E}S_I\left(u^i\right)$ when $X\left(u^i\right)$ is interior. To find $X'\left(u^i\right)$ use equation $(B.5)$ and the Implicit Function Theorem to get:

$$X'\left(u^i\right) = \frac{F\left(c^i\right)}{u'\left(X\left(u^i\right)\right) - \left(1 - F\left(c^i\right)\right)\vartheta'\left(X\left(u^i\right)\right)}.$$

Plugging this into the formula for $\frac{\partial \mathbb{E}S_I}{\partial u^i}$ reveals that the first restriction on the distribution of the shock, F, given in Proposition 11 is sufficient to make the marginal effect of u^i onto $\mathbb{E}S_I\left(u^i\right)$ strictly positive when $X\left(u^i\right)$ is interior. If instead there exists cases where $X\left(u^i\right) = 1$, then, in the region where $u^i \geq u_2$ the function $\frac{\partial \mathbb{E}S_I}{\partial u^i}$ simplifies to:

$$\frac{\partial \mathbb{E}S_I}{\partial u^i} = \lambda\left[F\left(c^i\right) - f\left(c^i\right)(v - \vartheta(1))\right]$$

and the second restriction on the distribution of the shock, F, given in Proposition 11 ensures it is always strictly positive, concluding the proof. ■

***Derivations for** Section 8.2.* Denote $p(c)$ the equilibrium market price at $t = 1$. Since the buyers' valuation is always positive, it holds that $p^b(c) > 0$ for all c. In principal trades, dealers bid up to a price that makes them indifferent whether or not to buy the asset at $t = 0$. Anticipating their choice to always sell at $t = 1$ for every non-negative price, they bid up to:

$$p^i = \mathbb{E}p^b(c) - \iota.$$

In agency trades, dealers only match sellers with buyers at $t = 1$, thus no transaction takes place at $t = 0$. Owners of bad assets sell their assets for any non-negative price at $t = 1$. Thus, they prefer principal trades as long as $\psi \geq \iota$. For owners of good assets, the expected payoff from selling via an agency trade is

$$\mathbb{E}\max\left\{p^b(c), u(x)\right\} - \psi.$$

Finally, the payoff from retaining the asset is $-\psi$ for owners of bad asset and $u(x) - \psi$ for owners of good assets. If $\psi \geq \iota$, the strategy profile can be described by the partition: $\{0, X, 1\}$. Good asset owners choose principal trades if $x \leq X$ and sell via agency trades otherwise. When X is interior, it must solve the indifference condition:

$$\mathbb{E}p^b(c) - \iota = \mathbb{E}\max\left\{p^b(c), u(X)\right\} - \psi$$

Clearly, when $\iota = 0$, the indifferent seller is the same as in the model analyzed in Section 5. Generally, the cutoff X is a function of ψ and ι, denoted $X(\psi, \iota)$, and it holds that $X(\psi, \iota) = X_0(\psi - \iota)$, where X_0 is the function of ψ described in the proof of Proposition 6. It follows that a marginal increase in ι has the same effect onto X that a marginal decrease in ψ has onto X_0. Expected volume of trade at $t = 1$ decreases since sellers opting for agency trades choose to retain for low realizations of c. Since buyers do not observe asset quality, there are gains from trade in the state at which the seller of good asset is indifferent whether to accept or reject the offer. Therefore, the marginal decrease in X reduces allocative efficiency. Since more sellers bear the delay cost, welfare also decreases unambiguously.

In the proof of Proposition 6, it is shown that if $\hat{c}(0) \geq \underline{c}$, there is a unique value $\underline{\psi}$ such that X_0 is zero at all $\psi \leq \underline{\psi}$, and $X_0(\psi)$ is continuous at $\underline{\psi}$. Thus, if $\psi - \iota \geq \underline{\psi}$, some owners of good assets choose principal trades, that is $X > 0$. As ι increases, X decreases until it is zero at $\bar{\iota} := \psi - \underline{\psi}$. For values of ι in the interval $\left[\psi - \underline{\psi}, \psi\right]$, only owners of bad assets opt for principal trades. When $\iota > \psi$, all sellers choose agency trades.

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