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Learning about the consumption risk exposure of firms ☆

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ABSTRACT

We structurally estimate an investment-based asset pricing model, in which firms' exposure to macroeconomic risk is unknown. Bayesian beliefs about this parameter are updated from firms' and industry peers' comovement between their productivity and consumption growth. The model implies that discount rates rise endogenously with the perceived risk exposure of firms, thereby depressing investment and valuation ratios. We test these predictions in the data and find strong support for them. We also confirm that cross-sectional learning from peers is crucial and that alternative Bayesian risk estimates, which ignore peer observations, do not predict firm variables.

1. Introduction

The consumption-based asset pricing paradigm states that risk premia arise from the comovement between consumption growth and returns. Despite its intuitive appeal, many early empirical tests did not find support for this prediction.[1] Inspired by a neoclassical investment model with a consumption-based pricing kernel and parameter uncertainty about firms' exposure to macroeconomic risk, we propose a novel cash flow beta of productivity to consumption growth based on Bayesian learning. We find strong support for a consumption-based pricing kernel, as the evolution of Bayesian beliefs about cash flow betas predicts investment rates, valuations ratios, and risk premia.

In our neoclassical investment model, firms' productivity is stochastic and correlated with consumption, rendering firms exposed to macroeconomic risk. We make two key assumptions about the parameter controlling the exposure of firms' productivity to consumption shocks. First, this parameter is constant over time but unknown. In response, agents learn about it through Bayesian updating. Second, firms in the same industry share the same exposure parameter. Because of the identical risk exposure among industry constituents, it is optimal

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[1] Recent papers that find support for the C-CAPM include Lettau and Ludvigson (2001) and Hansen et al. (2008).

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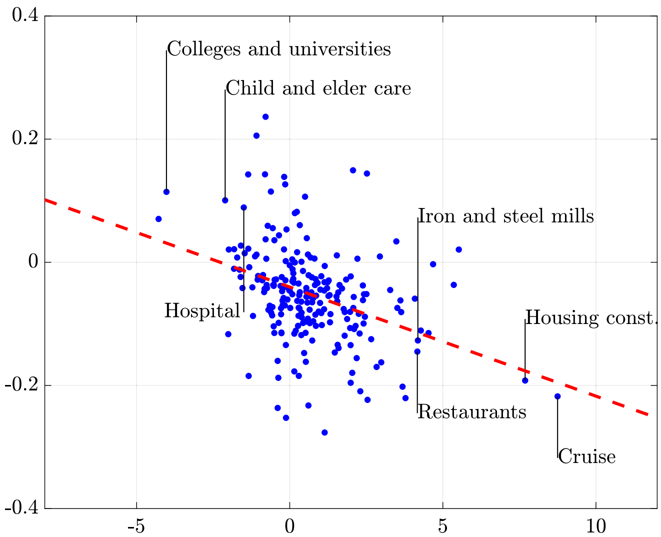

Fig. 1. Beliefs about Risk Exposure and Investment Rates. This figure illustrates the correlation between changes in the perceived risk exposure from 2006 to 2009 and changes in investment rates from 2007 to 2010 across industries, where investment rates are equally weighted within an industry.

for individual firms to incorporate peers' productivity as a signal into their learning about their own risk exposure. Using the collective observations of peers, firms thus learn from a rich source of cross-sectional information compared to the case in which they would learn solely from their own history.[2]

Our model features a consumption-based pricing kernel. As a result, Bayesian beliefs about the productivity exposure parameter affect firm decisions and characteristics. Intuitively, an increase in risk exposure beliefs upon the arrival of new information means that the perceived covariance between productivity and consumption has increased. Due to the consumption-based pricing kernel, this elevated covariance translates into higher risk premia, even though the true risk exposure is constant. Higher discount rates depress the value of new investment projects, thereby lowering investment rates and valuation ratios.

We test these predictions in the data by using panel regressions, and we find strong support for them. Specifically, we find that capital investment rates and valuation ratios, as measured by Tobin's Q, respond strongly negatively to the posterior mean risk exposure at the 1% significance level, when we control for other known cross-sectional determinants. These links are also economically significant. A one-standard-deviation increase in the mean risk exposure leads to, on average, a 5.6% decrease in investment and a 4.6% decrease in Tobin's Q.

To illustrate our results, we present in Fig. 1 the response of changes in investment rates from 2007 to 2010 to changes in the perceived risk exposure from 2006 to 2009 across industries. This period highlights the impact of learning on firm policies, as the large drop in consumption during the Great Recession caused a substantial revision in beliefs. The figure illustrates the negative response of firm investment to shifts in risk exposure beliefs. Investment rates fell significantly for industries with a surge in perceived risk exposure, such as cruise, housing construction, and iron and steel mills. In contrast, industries with a drop in risk exposure, such as colleges and universities, child and elder care, and hospitals, increased their investment.

In our model, the synchronous response of investment and valuation is driven by endogenous shifts in discount rates that learning about the risk exposure parameter induces. To test this conjecture, we employ both the implied cost of capital from accounting information[3] and realized stock returns as a proxy for discount rates. We find that both discount rate measures relate positively to the posterior mean risk exposure with a strong statistical significance. Economically, a one-standard-deviation rise in the mean risk exposure is accompanied by, on average, a 0.45% increase in the annualized implied cost of capital and a 0.71% increase in the realized return.

A key identification assumption underpinning our Bayesian learning is that firms learn from both their own history and industry peers. In the data, we identify each firm's peers using the SIC, NAICS, and Hoberg and Phillips (2016) text-based industry classification systems. We confirm that industry peers indeed have a commonality in firm-level productivity; the first principal component explains on average 35.9% of productivity variance within industries. Next, we illustrate the importance of industry peers by considering three alternative forms of learning. First, firms learn from their own history only; second, they learn from their own history plus randomly assigned peer firms; and third, they learn from industry peer observations only, ignoring their own history. We find that the first two approaches, which do not include peer observations, result in insignificant links between firm variables and risk exposure beliefs. In contrast, beliefs in the last approach strongly predict firm variables, similar to our baseline results, and thus highlight information spillovers across peer firms. In sum, all three experiments support the assumption that industry peers share the same risk exposure parameter.

Our neoclassical investment model features a decreasing returns to scale production function, convex capital adjustment costs, and Bayesian learning from peers about the risk exposure parameter. To quantitatively evaluate the model performance, we estimate with simulated method of moments the depreciation rate, capital share of production, adjustment cost parameter, idiosyncratic volatility, productivity exposure to consumption risk parameter, and price of risk in the pricing kernel. We identify these six parameters based on eight moments, which are the mean and variance of the investment rate, equity returns, Tobin's Q, and the posterior mean of risk exposure.

Overall, the model matches all moments well. Both in the model and data, investment rates average around 25% annually with a standard deviation of around 21%. In the data, stock returns are close to three times more volatile than investment rates, which the model can almost replicate. Specifically, stock returns have a volatility of 58% in the data, compared to 53% in the model. The model can match these moments with a large depreciation rate of 56%, an idiosyncratic volatility of 92%, and an adjustment cost parameter of 2.15. The large adjustment costs drive the wedge between the volatility of investment and stock returns. The magnitude of the adjustment costs parameter implies that firms spend around 6.4% of their output on capital adjustments. The model can also match firm-level average excess returns of 10% with a price of risk of 2.35.

The model also replicates fairly well an average Q of around 1.9 with a capital share of 0.75. Relative to the existing literature (e.g., Nikolov and Whited (2014)), our model also generates a volatile Tobin's Q. Intuitively, the time variation of valuation ratios such as Q reflects time variation in discount rates. Learning about risk exposure generates time variation in discount rates, even though the true parameter is constant. In our benchmark specification, each industry is assumed to have five firms to reflect the average number in the data. When the number of peers increases, the volatility of stock returns and Tobin's Q both decrease because the agent observes more signals from which to learn. Yet even with ten industry peers, our learning model significantly im-

[2] While we are not the first to measure cash flow betas (e.g., Bansal et al. (2005) and Da (2009)), our cash flow betas are distinctively based on Bayesian learning that uses the cross section of productivity growth among industry constituents.

[3] We measure the implied cost of capital following the approach of Hou et al. (2012).

proves on explaining the volatility of stock returns and Q compared to the literature, which has ignored parameter uncertainty.

Even though we did not target any regression coefficients in the estimation, the model can quantitatively generate the negative response of investment rates and Tobin's Q to risk exposure beliefs. The model also produces a positive relationship between risk exposure beliefs and the cost of capital, which resembles patterns in the data. In addition, we confirm that our model generates informational spillover effects. As captured in the data, beliefs about risk exposure driven only by industry peer observations negatively predict both investment and the valuation ratio and, simultaneously, positively predict the cost of capital.

To ensure robustness, we confirm that our main empirical findings hold, even when the true exposure to systematic risk is stochastic and even when there is uncertainty about the drift. Suppose that the true exposure changes over time, contrary to our baseline assumption. In this case, the learning-based estimate–derived from the constant-risk assumption–might misleadingly capture variations in true risk characteristics. To address this concern, we explicitly model the true risk exposure as an autoregressive process and estimate the beliefs distribution by using the Kalman filter. In this setting, we focus on ambiguity about the unconditional mean of systematic risk, the true value of which is constant by nature. In ways that mirror our baseline findings, the mean beliefs about this parameter still predict firm observables in our analysis.

Distinct from our focus on risk exposure, many prior studies have considered uncertainty with respect to the drift of productivity (e.g., Pastor and Veronesi (2003), Alti (2003), and Andrei et al. (2019)). In response, we extend our model and consider the joint learning about the drift and risk exposure of productivity. We confirm that beliefs about the drift predict firm observables, consistent with previous studies. More importantly, when we control for learning about the drift, we find that risk exposure beliefs are still powerful predictors of firm variables, similar to our baseline findings. Our analysis thus establishes that firms' real decisions and market valuation respond to learning about both systematic productivity risk and expected productivity growth.

The remainder of this paper is organized as follows. In Section 2, we describe the dynamics of aggregate consumption and firm-level productivity and derive the dynamics for Bayesian learning. In Section 3, we present empirical evidence that links beliefs about firms' risk exposure, investment, and valuation. We rationalize these empirical results by using a neoclassical investment model with cross-sectional learning in Section 4. In Section 5, we show that our empirical results are robust to a setting with time variation in the true risk exposure and learning about the productivity drift.

Literature Review

Our paper builds on the literature that studies parameter learning and its implications for asset valuations. Pastor and Veronesi (2009) provide a comprehensive review of learning models in finance. Jovanovic and Nyarko (1994), David (1997), Weitzman (2007), Collin-Dufresne et al. (2016), and Johannes et al. (2016) show that learning about parameters governing the economy generates regularities in asset returns and business cycles, which otherwise seem puzzling. Other papers study learning about an unknown aggregate state[4] or endogenous information acquisition, such as Veldkamp (2006). Focusing on the aggregate implications, however, these studies have not examined how learning affects the cross section of corporate valuations and investment, which is the goal of our paper.

The Bayesian learning proposed in this paper is related to the limited attention literature. As a particular form of non-Bayesian learning arising from limited attention, Malmendier and Nagel (2016) and Nagel and Xu (2022) document learning with fading memory. Both forms of learning share the same insight that past observations shape the beliefs about parameters affecting asset valuations. In contrast to Bayesian learning, agents with fading memory put more weight on recent observations and thus the influence of past data on beliefs gradually fades over time.

Our paper is also closely related to prior studies that highlight uncertainty about a firm-level parameter, such as dividend growth (Veronesi (2000)), mean productivity (Pastor and Veronesi (2003); Alti (2003)), return-to-scale in the production function (Johnson (2007)), and mean cash flow (Andrei et al. (2019)). Distinctive from these studies, we focus on ambiguity with respect to exposure to macroeconomic risk, similar to the idea of Ai et al. (2018) and Li et al. (2023). We complement these recent studies by elaborating upon the learning mechanism. In our model, agents learn from the history of realized productivity instead of noisy independent signals, as is the case in prior studies. Furthermore, we propose learning from peer observations, which results in unique information spillovers.

The idea of learning from peers is related to Foucault and Fresard (2014), who document firm investment responding to peers' Tobin's Q. In their model, managers learn from peers' stock prices because demand is correlated across firms and investors trade based on private information. In the absence of private information, we rationalize peer learning with a neoclassical investment model, where firms in the same industry share the same risk exposure parameter. Relatedly, informational spillover effects on earnings announcement days have been found by Patton and Verardo (2012) and Savor and Wilson (2016).[5]

This study is also related to the literature on consumption-based asset pricing, including studies by Bansal et al. (2005), Da (2009), and Boguth and Kuehn (2013). These studies reveal that the cross-sectional dispersion in expected returns is driven by the comovement between consumption growth and securities' cash flows. Distinct from these prior studies, we consider Bayesian learning about risk exposure from the comovement between productivity and consumption growth and expand the implications of consumption risk to the time series dimension.

More broadly, our paper is also related to dynamic investment models, which examine the implications of firms' optimal decisions for asset returns. Prior studies, including Berk et al. (1999), Gomes et al. (2003), Carlson et al. (2004), Zhang (2005), and Kuehn and Schmid (2014), all show that observed patterns in stock and bond returns emerge as a result of corporate investment policy. We complement this literature by establishing new regularities about investment and returns caused by parameter learning. In our measurements of firm investment, Tobin's Q, and firm-level productivity, we explicitly account for the role of intangible assets. We measure the intangible components by following Eisfeldt and Papanikolaou (2013) and Peters and Taylor (2017).

Finally, our structural estimation follows Nikolov and Whited (2014) and Hennessy and Whited (2007), who apply the simulated method of moments to dynamic models. While these prior studies focus mainly on the impact of financing frictions on firm investment, we consider optimal firm policies under parameter uncertainty.

2. Bayesian learning about systematic risk

In this section, we describe the dynamics of aggregate consumption and firm-level productivity. Firms are exposed to aggregate consumption risk via their productivity process. Importantly, the exposure to consumption risk is unknown and must be learned over time. Firms observe their own productivity growth and aggregate consumption growth. Moreover, firms observe productivity growth of their peers in the same industry as signals. This set of signals is informative because

[4] These include Veldkamp (2005), Lettau et al. (2007), Ai (2010), Boguth and Kuehn (2013), and Croce et al. (2014).

[5] In a different context, Boguth and Kuehn (2013) and Jurado et al. (2015) underscore the importance of using a cross section of signals that share a common truth. We extend this idea to the context of firm investment and show that cross-sectional information is crucial for optimal corporate decisions.

firms in the same industry share the same exposure to consumption risk. Based on this setup, we derive the dynamics for the Bayesian beliefs about the risk exposure parameter.

2.1. Dynamics

Aggregate consumption growth $g_{c,t+1}$ is normally distributed with drift μ and volatility σ_c and given by

$$g_{c,t+1} = \mu + \sigma_c \eta_{t+1}, \tag{1}$$

where η_{t+1} is an i.i.d. standard normal innovation. Even though the specification for consumption growth does not allow for time varying uncertainty, our empirical results are not affected by this assumption because Bayesian learning requires only demeaned consumption growth as an input. As in Kuehn and Schmid (2014), firms' productivity is stochastic and correlated with consumption, rendering firms exposed to macroeconomic risk. Specifically, firm i productivity growth $g_{i,t+1}$ is a mixture of an idiosyncratic and aggregate shock and given by

$$g_{i,t+1} = \mu + b\sigma_c \eta_{t+1} + \sigma \varepsilon_{i,t+1}, \tag{2}$$

where $\varepsilon_{i,t+1}$ is a firm-specific i.i.d. standard normal innovation, σ quantifies the magnitude of idiosyncratic risk, and b controls the exposure of productivity to aggregate risk.

The risk exposure parameter b is our main focus. Intuitively, an increase in b amplifies the covariance between productivity and consumption and results in productivity displaying more systematic risk. In a consumption-based asset pricing framework, this elevated covariance translates into higher risk premia, thereby depressing optimal investment. We explore these theoretical channels in detail in Section 4.

We make two key assumptions about the risk exposure parameter. First, the parameter b is constant over time but unknown. In response, decision makers learn about it through Bayesian updating. Second, firms in the same industry share the same exposure parameter. These industry peers become heterogeneous ex-post due to idiosyncratic productivity shocks, but they share a common characteristic.

A limitation of our specification is that the risk exposure does not change over time. This implies that in the long run, agents can perfectly learn this parameter. In Section 5.1, we confirm that our main findings hold, even when the true exposure to systematic risk is dynamic. We also provide empirical evidence that justifies our assumption that industry peers share the same risk exposure parameter; we discuss this evidence in Section 3.7 by considering three alternative forms of learning.

2.2. Learning

In this section, we derive the Bayesian beliefs about the risk exposure parameter b. Agents are equipped with prior beliefs about the parameter b, which are normally distributed with mean $m_{b,0}$ and standard deviation $\sigma_{b,0}$. Thereafter, they receive new information: the realized productivity of every industry constituent and consumption growth. Because of the identical risk exposure among industry constituents, peers' productivity should be informative with respect to each other's risk exposure. Recognizing this, agents refer to their peers' collective observations in updating their parameter beliefs. Nevertheless, learning about b is nontrivial because productivity is subject to unobservable idiosyncratic shocks, which agents cannot distinguish from the systematic component.

To formulate the learning process, we let g_t denote the $n_t \times 1$ vector of productivity growth for n_t constituents of a specific industry at time t.[6] Conditional on the observations of productivity and consumption, beliefs about the parameter are revised according to Bayes' law. This learning mechanism induces a recursive structure of the posterior distribution

$$\text{Prob}(b|g_1, .., g_t, g_{c,1}, .., g_{c,t})$$
$$\propto \text{Prob}(g_t|b, g_{c,t}) \times \text{Prob}(b|g_1, .., g_{t-1}, g_{c,1}, .., g_{c,t-1}),$$

where we use the fact that g_t depends only on current consumption growth and risk exposure. Since we assumed a Gaussian prior, the posterior distribution remains normal with mean $m_{b,t}$ and standard deviation $\sigma_{b,t}$. As we show in Appendix A, the conditional mean and standard deviation of beliefs follow a recursive structure given by

$$m_{b,t} = (1 - \kappa_t \sigma_{b,t}^2) m_{b,t-1} + \kappa_t \sigma_{b,t}^2 \hat{b}_t \tag{3}$$

$$\frac{1}{\sigma_{b,t}^2} = \frac{1}{\sigma_{b,t-1}^2} + \kappa_t, \tag{4}$$

where $\kappa_t = n_t \eta_t^2 \sigma_c^2 / \sigma^2 \geq 0$ is the scaled consumption shock, and $\hat{b}_t$ measures the covariance between consumption and productivity shocks based on time-t observations only, i.e., $\hat{b}_t = \left[\sigma_c \eta_t \sum_{i=1}^{n} (g_{i,t} - \mu)\right] / (n_t \sigma_c^2 \eta_t^2)$.[7]

In belief updates, the posterior mean $m_{b,t}$ is a weighted average of the prior mean $m_{b,t-1}$ and the sample estimate $\hat{b}_t$, with the weights determined by the parameter uncertainty (measured by $\sigma_{b,t}^2$) and the informativeness of the data (measured by κ_t).[8] The revision is more sensitive to new observations when agents are more uncertain about the parameter (i.e., when $\sigma_{b,t}$ is high), and also when the new data has higher informativeness κ_t. The informativeness improves when there are more firms to learn from and when the ratio of consumption to productivity volatility is larger. Lastly, the precision of beliefs, $1/\sigma_{b,t}$, increases monotonically over time.

In our formulation, each firm's own productivity and peers' observations constitute sources of learning by determining the sample estimate $\hat{b}_t$. Specifically, this updating places equal weights between a firm's own observation and each of its peers. In a more generalized setting in which accounting noise makes observations from peers less precise compared to a firm's own, the learning mechanism would assign different weights. In this case, the Bayesian update imposes more weight on the accurate signal (own productivity) than it does on the inaccurate signal (peers' productivity).

2.3. Measurement

To measure productivity growth in the data, we assume that firms employ a decreasing-returns-to-scale production technology using capital as input. The profit $Y_{i,t}$ of firm i at time t is given by

$$Y_{i,t} = X_{i,t}^{1-\alpha} K_{i,t}^{\alpha}, \tag{5}$$

where $X_{i,t}$ denotes the level of productivity, $K_{i,t}$ the capital stock, and $0 < \alpha < 1$ is the capital share of the production. Productivity follows a random walk $X_{i,t} = X_{i,t-1} e^{g_{i,t}}$, for which its growth $g_{i,t+1}$ is given in equation (2). Given data on profit and capital, the neoclassical production function implies that log productivity growth can be computed as

$$g_{i,t} = \frac{g_{y,i,t} - \alpha g_{k,i,t}}{1 - \alpha}, \tag{6}$$

[6] To save on notation, we do not include an index for industries.

[7] These updating equations are based on the sampling probability that assigns equal weight to all past observations. As an alternative to Bayesian learning, we consider learning with fading memory, in which more recent observations receive greater weight. The belief updates for this alternative form of parameter learning are provided in Internet Appendix E.

[8] With more observations over time, the posterior mean $m_{b,t}$ converges to the ordinary least squares estimate in the regression of productivity onto consumption growth. The comparison between Bayesian and frequentist approaches is provided in Internet Appendix D.

Table 1
Summary Statistics. This table presents descriptive statistics based on the merged CRSP-Compustat sample from 1964 to 2021. In Panel A, the investment rate is the ratio of total investment (tangible plus intangible investment) to lagged total capital (tangible plus intangible capital stock). Q is the market value of assets divided by total capital. The implied cost of capital (ICC) is measured from accounting information and excess returns from CRSP. Size is defined as the logarithm of a firm's total assets. Profitability is income before extraordinary items divided by lagged total assets. Leverage is the ratio of book debt to the market value of assets. Firm age is the logarithm of the number of years since a firm's stock price first appeared in CRSP. In Panel B, $m_{b,t}$ is the mean and $\sigma_{b,t}$ the standard deviation of the distributions of risk exposure beliefs. The belief distributions are estimated based on observations of industry peers identified by either SIC, NAICS, or the text-based industry classification (TNIC). The last column reports the within-industry dispersion of variables, which are the target of our structural estimation.

Panel A: Firm Variables

	Across Firms					Within Industry
Variable	Mean	Std. Dev.	25%	50%	75%	Std. Dev.
Investment rate	0.247	0.217	0.136	0.206	0.298	0.213
Q	1.940	2.349	0.871	1.307	2.123	2.295
ICC	0.046	0.279	-0.015	0.016	0.057	0.277
Excess returns	0.099	0.577	-0.231	0.015	0.301	0.576
Size	1.330	2.106	-0.227	1.226	2.797	1.853
Profitability	0.027	0.141	0.001	0.046	0.090	0.139
Leverage	0.323	0.309	0.062	0.232	0.499	0.283
Age	2.621	0.614	2.079	2.639	3.091	0.598
Market cap.	5.096	2.325	3.338	4.980	6.749	2.147

Panel B: Bayesian Beliefs about Risk Exposure

	Across Firms					Within Industry
Variable	Mean	Std. Dev.	25%	50%	75%	Std. Dev.
$m_{b,t}$ (SIC)	3.632	3.866	1.399	3.239	5.585	1.837
$m_{b,t}$ (NAICS)	3.744	3.923	1.281	3.401	6.033	2.261
$m_{b,t}$ (TNIC)	5.885	8.075	0.659	4.699	9.761	6.811
$1/\sigma_{b,t}$ (SIC)	0.316	0.097	0.245	0.287	0.357	0.041
$1/\sigma_{b,t}$ (NAICS)	0.359	0.125	0.258	0.334	0.426	0.053
$1/\sigma_{b,t}$ (TNIC)	0.503	0.258	0.312	0.424	0.614	0.077

where $g_{y,i,t}$ denotes the log growth rate of profits, and $g_{k,i,t}$ denotes the log growth rate of capital of firm i. We use the same functional form for profit in the empirical exercise in Section 3 and in the model in Section 4.

3. Empirical evidence

In this section, we present the empirical evidence linking beliefs about firms' risk exposure, investment, and valuation. These empirical results are rationalized by a neoclassical investment model with cross-sectional learning in Section 4.

3.1. Data

Our data consist of annual observations for non-financial and non-utility firms from the merged CRSP-Compustat dataset for the years 1964 to 2021. Among firm-year observations, we exclude data points with negative or missing values for sales, total assets, or the net value of property, plant, and equipment. Also, we exclude observations with missing stock prices or returns within a year. Our final dataset includes 121,394 firm-year observations. In Table 1, we report summary statistics of the variables employed in our analysis in Panel A.

The estimation of firms' risk exposure requires data on firm-level productivity (6) and consumption growth. To measure firm-level profits, we use sales (SALE) minus the costs of goods sold (COGS), as in Imrohoroglu and Tuzel (2014) and Ai et al. (2020). The total capital stock is the sum of tangible and intangible capital. The tangible component is given by the net value of property, plant, and equipment (PPENT), and the intangible one is estimated by Peters and Taylor (2017). We adjust the growth rates for profit and capital for inflation by using the GDP deflator. We also lag the growth rate of capital by one year to account for one-period time-to-build as in the neoclassical investment model. To measure consumption, we use the real per capita growth rate of nondurable goods and services expenditures.

Following Peters and Taylor (2017), the investment rate is the ratio of total investment divided by the lagged value of total capital. Similarly, Tobin's Q is the market value of assets divided by total capital.[9] Total investment is the sum of intangible and tangible investment. Intangible investment equals research and development expense (XRD) plus 30% of SG&A expenses, which are given by (XSGA) minus (XRD) minus (RDIP). As in Bai et al. (2019), we measure the amount of tangible investment as a change in the net value of property, plant, and equipment (PPENT) plus depreciation, which is computed as depreciation (DP) minus amortization (AM). Regression control variables include firm size, profitability, leverage, firm age, and market capitalization.[10]

[9] Our baseline measurement of investment rates and Q requires the variable PPENT; however, its value is missing for 23% of firm-year observations in the merged CRSP-Compustat dataset. In Internet Appendix B, we extend the sample size by utilizing total assets (AT) when PPENT is missing and confirm that our empirical results remain significant.

[10] Firm size is defined as the logarithm of total assets (AT). Profitability is income before extraordinary items (IB) divided by lagged total assets. Leverage is the ratio of book debt to the market value of assets. Book debt is the sum of debt in current liabilities (DLC) and long-term debt (DLTT). The market value of assets is book debt plus the market capitalization equity, which is the product of common shares outstanding (CSHO) and the stock price (PRCC). Firm age is the logarithm of the number of years since the firm's stock price first appeared in CRSP.

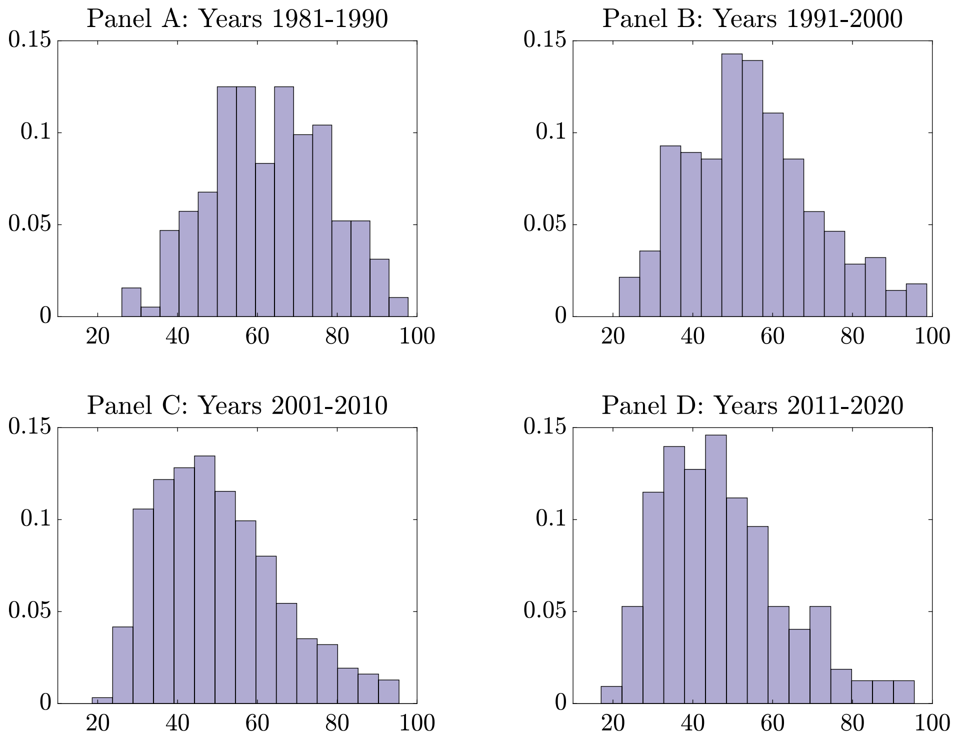

Fig. 2. Common Fluctuations in Productivity. This figure shows the histogram of the explained variance (in percent) generated by the common fluctuation of productivity among industry peers. The common fluctuation for each industry is identified by the first principal component using subsamples of 10 years of data.

We measure firms' implied cost of capital using accounting data as in Hou et al. (2012). Specifically, the cost of capital is the particular discount rate that makes the present value of expected future cash flows equal to the firms' market value. The future cash flow is forecast by a cross-sectional model that relates firms' earnings to other accounting variables, such as total assets, dividends, and accruals. In addition, as an alternative measure of the cost of capital, we use realized stock returns over the risk-free rate, the latter obtained from Kenneth French's website. As is common in the literature, we compute realized returns from July of the corresponding year until June of the following year. Both the implied cost of capital and realized returns are annualized.

We follow the literature and identify each firm's peers using the industry classifications SIC or NAICS (e.g., Kahle and Walking (1996) and Krishnan and Press (2003)). We begin with four-digit classifications of each system. If a firm has too few peers at this granular level of industry, we relax the definition of peers to ensure a sufficient number of cross-sectional observations for learning. Specifically, for four-digit industries that have fewer than five constituents at any point in time, we expand the reference set to include firms in the same three-digit industries throughout the course of learning.[11] As an alternative definition of industries, we employ the text-based classification recently developed by Hoberg and Phillips (2010). This classification is based on product similarity among firms as measured through a text-based analysis of 10-K filings. This text-based network industry classification system (hereafter, referred to as TNIC) is obtained from the Hoberg-Phillips Data Library.

3.2. Commonality in firm-level productivity

Assuming a capital share in production α of 0.65 as in Cooper and Ejarque (2003) and given data on real profit and capital growth, we can compute productivity growth (6) for each firm-year. A key premise underlying our analysis is that industry peers have common fluctuations in productivity. This section examines whether such commonality among peers is indeed present in the data. To this end, we conduct a principal component analysis on the dataset of industry constituents' productivity.[12] This analysis produces the first principal component, which we interpret as representing the common fluctuation. We then focus on the share of variance explained by the first component to assess the significance of the common fluctuation.

Fig. 2 shows the histogram of the percentage of productivity variance explained by each industry's common fluctuation, which is identified for each 10-year subperiod. We find that the median fraction of firm-level variability explained by the first components ranges from 31.5% to 45.0% across subperiods. When all estimates across subperiods are pooled, the common fluctuations explain on average 35.9% of productivity variance. This result supports the assumption that a common factor is present in firm-level productivity, and therefore peers' observations are informative of each other's risk exposure.

At the same time, the residual variance not explained by the first component is non-trivial, suggesting that idiosyncratic innovations constitute another sizable share of productivity variability. In learning about the common risk exposure parameter, the idiosyncratic component adds substantial noise to productivity observations. Therefore, we expect that the formation of precise beliefs about risk exposure will require many observations.

In addition, we confirm that the common fluctuation in productivity highly correlates with aggregate consumption. Using a principal component analysis, we identify the common trend in productivity by computing the equal-weighted average across industries of their respective first principal components. The resulting time series of economy-wide

[11] In an alternative analysis, we use four-digit SIC and NAICS industries without relaxing the definition of peers. Our main empirical findings continue to hold when we use this alternative definition of peers.

[12] This analysis requires balanced panel data; consequently, we must subsample firms with no missing observations during the entire analysis period. To ensure sufficient observations for each industry, we shorten the estimation window to 10 years and conduct the analysis for each of the 10-year subperiods.

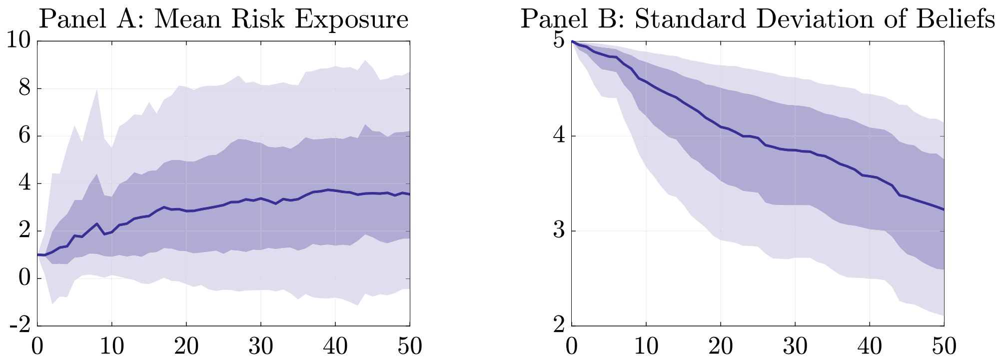

Fig. 3. Cross-Sectional Distribution of Risk Exposures. This figure depicts the cross-sectional distribution of mean risk exposure $m_{b,t}$ (Panel A) and the beliefs' standard deviation $\sigma_{b,t}$ (Panel B) across industries as a function of industry age. The solid line is the median of the distribution for each age. The dark shaded area indicates the distribution between the 25th and 75th percentiles and the light shaded area the distribution between the 10th and 90th percentiles.

productivity correlates significantly with consumption, as the correlation coefficient ranges from 0.42 to 0.80 across subperiods. This high correlation coefficient lends empirical support to our formulation of the productivity process, where productivity loads on aggregate consumption, leading to common fluctuations.

3.3. Beliefs

Given our estimates for productivity growth, we are now equipped to estimate Bayesian beliefs about risk exposure formulated in equations (3) and (4). To do so, we measure idiosyncratic volatility σ for each industry as the standard deviation of the residuals in the regression of firm-level productivity growth onto consumption growth.[13] Importantly, while Bayesian learning requires the measurement of idiosyncratic risk, it does not depend on an estimate of consumption risk.[14] We set the productivity drift μ equal to the consumption drift, which is in line with balanced growth in the economy.[15]

Bayesian learning requires a date-0 prior belief. We assume a diffuse prior by setting $m_{b,0}$ to 1 and $\sigma_{b,0}$ to 5.[16] This large dispersion represents decision makers' ambiguity with respect to risk exposure at the beginning of the learning process. In addition, we skip the first 5 years of beliefs so that date-0 prior's impact on the results is muted.[17] After initializing the belief distribution, we update the distribution $m_{b,t}$ and $\sigma_{b,t}$ for each industry using the cross-section of firm-level productivity.

Prior to delving into our regression analysis, we provide summary statistics regarding the distribution of beliefs in Panel B of Table 1. The average estimates of risk exposure, denoted as $m_{b,t}$, exhibit a variation from 3.63 to 5.89, contingent on the industry classification. Similarly, the precision of beliefs, represented as the inverse of the beliefs' standard deviation $\frac{1}{\sigma_{b,t}}$, shows a mean value that varies between 0.32 and 0.50, also dependent on the particular industry classification.

In Fig. 3, we present the evolution of the cross-sectional distribution of mean risk exposure $m_{b,t}$ (Panel A) and the standard deviation of beliefs $\sigma_{b,t}$ (Panel B) across industries, as a function of industry age. Panel A illustrates the dynamics of the cross-sectional variance in mean risk exposures over a span of more than 50 years of parameter learning. The initial risk exposure estimate for each industry begins at a prior value of $m_{b,0} = 1$. In the early years, the cross-sectional dispersion in the mean risk exposures widens due to the idiosyncratic character of each industry's parameter learning process. However, with the accumulation of observations over time, the mean risk exposure estimates gradually settle to form a steady distribution. Panel B depicts the refining precision of beliefs for each industry over time. The standard deviation of beliefs for each industry, which initially starts from the prior value of $\sigma_{b,0} = 5$, decreases as more observations become available over time, thus demonstrating an increase in the precision of beliefs.

In our following analysis, we ignore the impact of beliefs' standard deviation on firm variables. Because the true risk exposure parameter is assumed to be constant, the beliefs' dispersion is a monotonically declining process. As a result, the dynamics do not capture interesting cross-sectional patterns.

3.4. Firm investment

In this section, we test whether firms' investment policies respond to their estimated risk exposures. Intuitively, as the estimated risk exposure increases, firms are more exposed to aggregate productivity risk, which raises their discount rate and thus lowers the NPV of new investment projects. As a result, we expect a negative relationship between investment rates and risk exposure.

We test this hypothesis by running panel regressions of the form

$$\frac{I_{i,t}}{K_{i,t}} = \alpha_I + \beta \times m_{b,t-1} + \gamma \times \text{Controls}_{i,t-1} + \epsilon_{i,t}, \tag{7}$$

where $I_{i,t}/K_{i,t}$ denotes firm i's investment rate, and α_I an industry fixed effect. Pastor et al. (2017) point out that a firm fixed effect is designed to capture the time series response of explanatory variables. We choose the industry fixed effect over the firm fixed effect because the latter might introduce a spurious regression bias, as studied in Ferson et al. (2003), for firms with few data points.[18] With parameter beliefs being updated over time, we expected firm responses to be particularly pronounced in the time-series dimension. The controls include variables that have

[13] Our baseline estimation assumes that each industry's idiosyncratic volatility σ is constant over time. In practice, however, the volatility may fluctuate and therefore affect the belief updates specified in equations (3) and (4). In Internet Appendix A, we show that our results are robust to this concern. Specifically, we estimate each industry's conditional volatility σ_t and update risk exposure beliefs accordingly. We find that our empirical results are robust to the heteroskedasticity of idiosyncratic shocks.

[14] In the belief dynamics specified in equations (3) and (4), the aggregate signal and informativeness depend on $\sigma_c \eta_t$, which is identical to demeaned consumption growth $g_{c,t} - \mu$. Importantly, demeaned consumption growth can be identified without specifying the dynamics of consumption volatility.

[15] In section 5.2, we relax this assumption and formulate the parameter learning when the productivity drift is specific to each industry and also uncertain.

[16] To ensure robustness, we alternatively choose different values ranging from 0.5 to 2 for $m_{b,0}$ and from 3 to 10 for $\sigma_{b,0}$; when we do so, we find that our main empirical findings continue to hold.

[17] For robustness, we conduct an alternative estimation in Internet Appendix C, in which we skip the first ten years of beliefs. Our empirical results continue to hold for this longer burn-in period.

[18] In alternative specifications that replace the industry fixed effect with a firm fixed effect, we confirm that all of our regression results continue to hold with a similar significance.

Table 2

Risk Exposure Beliefs and Investment. This table presents panel regressions of investment rates on risk exposure beliefs and controls. The investment rate is the ratio of total investment (tangible plus intangible investment) to lagged total capital (tangible plus intangible capital stock). Mean risk exposure beliefs $m_{b,t}$ are calculated from cross-sectional observations of industry constituents, which are identified based on SIC or NAICS codes or the text-based classification system TNIC. The controls are size, age, profitability, leverage, and Q. Industry fixed effects are included in all but specification (5), where only year fixed effects are included. Specifications (6) include both industry and year fixed effects. Standard errors are clustered by firms. t-statistics are presented in parentheses below the parameter estimates. *, **, and *** denote statistical significance at the 10%, 5%, and 1% levels, respectively.

Specification Industry class.	(1) SIC	(2) NAICS	(3) TNIC	(4) SIC	(5) SIC	(6) SIC
$m_{b,t-1}$	-0.0035*** (-7.36)	-0.0036*** (-7.31)	-0.0017*** (-11.06)	-0.0032*** (-7.15)	-0.0019*** (-6.70)	-0.0016*** (-3.28)
Size	-0.0103*** (-17.37)	-0.097*** (-16.07)	-0.0088*** (-11.64)	-0.0115*** (-19.24)	-0.0061*** (-11.10)	-0.0077*** (-12.97)
Age	-0.0028*** (-32.88)	-0.0029*** (-32.84)	-0.0027*** (-28.87)	-0.0025*** (-30.50)	-0.0029*** (-29.13)	-0.0024*** (-24.95)
Profitability	0.2496*** (24.49)	0.2427*** (23.76)	0.2566*** (22.18)	0.1562*** (18.20)	0.2058*** (20.99)	0.2295*** (22.94)
Leverage	-0.1076*** (-20.55)	-0.1078*** (-19.66)	-0.1067*** (-17.17)	-0.0795*** (-15.64)	-0.1313*** (-25.08)	-0.1185*** (-22.64)
Q				0.0258*** (26.82)		
Industry FE	Yes	Yes	Yes	Yes	No	Yes
Year FE	No	No	No	No	Yes	Yes
N	112,155	107,131	72,634	112,155	112,155	112,155
adj. R^2	0.117	0.113	0.139	0.191	0.100	0.137

been found to affect investment, namely firms' size, age, profitability, leverage, and Tobin's Q.[19]

We report our regression results in Table 2. In specification (1), firms' peers are identified by SIC codes. The mean risk exposure $m_{b,t}$ derived from these peers' observation forecasts investment with strong statistical significance, as documented by a t-statistic of -7.36. This time-series association suggests that when beliefs about risk exposure are revised upward (downward), firms reduce (raise) investment.

This negative connection persists under alternative industry classifications. In specifications (2) and (3), we refer to NAICS or TNIC instead of SIC codes to identify industry peers. The resulting $m_{b,t}$ obtained from these alternative peers continues to be a significant and negative predictor of investment at the 1% level. Economically, a rise in the mean risk exposure by one standard deviation is accompanied by, on average, a 5.6% decrease in investment (i.e., the annual investment rate changes from 0.247 to 0.233).

In specification (4), we find that $m_{b,t}$ continues to affect investment, even after controlling for Q. This finding suggests that risk exposure beliefs reveal firms' risk characteristics beyond what Tobin's Q represents. This additional informativeness of $m_{b,t}$ is consistent with our structural model, in which the estimated parameters indicate decreasing returns to scale in production, resulting in a divergence between marginal and average Q. Consequently, state variables affecting marginal Q, such as risk exposure beliefs, convey additional information to which firm investment responds.

Beyond the temporal response, $m_{b,t}$ appears to explain the cross section of investment as well. In specification (5), we include a year fixed effect and find a strongly negative coefficient on $m_{b,t}$. This result shows that at a given point in time, firms with greater risk exposure invest less. This cross-sectional evidence further testifies to the impact of risk exposure beliefs on firm policy. As a robustness test, we include both industry and time-fixed effects in specification (6). We find that $m_{b,t}$ continues to negatively predict the investment rate.

It is worth noting that fluctuations in our risk estimate are only attributable to learning, while the true parameter is assumed to be constant. In contrast to this assumption, one might argue that the true risk exposure itself possibly changes over time. In such a case, our learning-based risk estimate might misleadingly capture variations in the true parameter. We address this concern in Section 5.1 by extending our model to consider parameter learning when the true risk exposure is dynamic. There, we show that beliefs about the unconditional mean risk exposure still predict empirical investment.

3.5. *Firm valuation*

Considering that risk exposure is a fundamental characteristic, we expect that learning about this parameter will also influence other firm variables beyond investment. Here, we consider the firms' market valuation. If market participants engage in learning about risk exposure, the market value of firms compared to their book value is likely to respond to updates in parameter beliefs. More specifically, a rising risk exposure raises discount rates and thus depresses valuations.

To examine this link empirically, we conduct a panel regression of the form

$$Q_{i,t} = \alpha_I + \beta \times m_{b,t} + \gamma \times \text{Controls}_{i,t} + \epsilon_{i,t}, \tag{8}$$

where $Q_{i,t}$ represents the valuation ratio, and α_I an industry fixed effect. Control variables are firm size, age, profitability, and leverage. This regression specification is similar to that of Pastor and Veronesi (2003), except that we additionally include risk exposure beliefs.[20] In this analysis, we test the contemporaneous link between valuation and

[19] Gala et al. (2020) find that firm size and cash flow (roughly equivalent to our measure of profitability) contain information about the marginal value of capital beyond what average Q conveys. Also, investment is affected by agency conflicts associated with leverage (e.g., Myers (1977)). Investment opportunities often change with firms' age, as discussed in Adelino et al. (2017).

[20] Pastor and Veronesi (2003) propose a model that predicts the market-to-book ratio of equity. In our model, firms' financing choices are not considered, so Q represents the valuation ratio of total firm value.

Table 3
Risk Exposure Beliefs and Q. This table presents panel regressions of Q on risk exposure beliefs and controls. Mean risk exposure beliefs $m_{b,t}$ are calculated from cross-sectional observations of industry constituents, which are identified based on SIC or NAICS codes or the text-based classification system TNIC. The controls are size, age, profitability, and leverage. Industry fixed effects are included in all but specification (4), where only year fixed effects are included. Specification (5) includes both industry and year fixed effects. Standard errors are clustered by firms. t-statistics are presented in parentheses below the parameter estimates. *, **, and *** denote statistical significance at the 10%, 5%, and 1% levels, respectively.

Specification Industry class.	(1) SIC	(2) NAICS	(3) TNIC	(4) SIC	(5) SIC
$m_{b,t}$	-0.0176***	-0.0208**	-0.0149***	-0.0001	-0.0278***
	(-2.80)	(-3.45)	(-5.03)	(-0.02)	(-4.54)
Size	0.0497***	0.0445***	0.0806***	0.0248**	0.0412***
	(4.76)	(4.16)	(6.10)	(2.61)	(3.90)
Age	-0.0117***	-0.0147***	-0.0150***	-0.0292***	-0.02226***
	(-8.64)	(-10.45)	(-9.51)	(-17.79)	(-14.45)
Profitability	3.3459***	3.2981***	3.8931***	3.4656***	3.6701***
	(19.04)	(18.47)	(17.35)	(18.71)	(20.35)
Leverage	-0.9771***	-1.0341***	-1.0806***	-1.1617***	-0.9378***
	(-10.78)	(-10.81)	(-9.93)	(-11.56)	(-10.31)
Industry FE	Yes	Yes	Yes	No	Yes
Year FE	No	No	No	Yes	Yes
N	121,410	115,715	76,308	121,410	121,410
adj. R^2	0.130	0.128	0.156	0.096	0.160

parameter beliefs. Unlike investment, which is a flow variable, firm values are measured by a snapshot of time, and thus parameter beliefs do not have to be lagged.

Table 3 reports the regression results. We find that Q is strongly connected to risk exposure beliefs. In specifications (1) to (3), the mean risk exposure negatively predicts Q at the 1% level across industry classifications. Economically, the coefficient estimates indicate that a rise in the mean exposure by one standard deviation is associated with an average 4.64% decline in the valuation ratio (i.e., Q changes from 1.940 to 1.850).

In specification (4), we include a year fixed effect only and find that $m_{b,t}$ becomes insignificant. However, $m_{b,t}$ turns strongly significant in specification (5), which includes both year and industry fixed effects. We, therefore, conclude that the negative link between risk exposure beliefs and Q holds even when relevant trends common to all firms are taken into account.

So far, we have reported that both investment and valuation respond strongly to risk exposure beliefs. We conjecture that these synchronous reactions are driven by shifts in firms' discount rates, which the parameter learning induces. In the following sections, we test this hypothesis.

3.6. Cost of capital

In a consumption-based asset pricing model, firms' cash flow exposure to consumption risk is a crucial determinant of their discount rates. If the risk exposure parameter is unknown, belief dynamics about this parameter will affect discount rates. In this section, we test this hypothesis by examining the direct link between risk exposure beliefs and discount rates. This analysis requires the measurement of discount rates, and we use both the implied cost of capital and realized stock returns as the proxy.

We examine the contemporaneous relation between the implied cost of capital and risk exposure beliefs by conducting Fama-MacBeth regressions. Specifically, we cross-sectionally regress the implied cost of capital on risk exposures and other firm characteristics

$$\text{ICC}_{i,t} = \alpha_t + \beta_t \times m_{b,t} + \gamma_t \times \text{Controls}_{i,t} + \epsilon_{i,t}, \tag{9}$$

where $\text{ICC}_{i,t}$ denotes the annual estimate of the implied cost of capital. As in Hou et al. (2015), control variables are the log market capitalization, investment rate, and profitability.

In Table 4 Panel A, specifications (1) through (3) present the results of univariate regressions on the mean risk exposure. Importantly, the risk exposure belief is strongly positively related to the implied cost of capital across different industry classifications at the 1% or 5% significance level. Economically, a one-standard-deviation increase in the mean risk exposure leads to a rise in the implied cost of capital by 0.45% per year, on average. We note that this rise in the cost of capital is consistent with the response of investment and valuation, which both decrease with $m_{b,t}$. In addition, when other risk characteristics are taken into account in specifications (4) through (6), risk exposure beliefs continue to associate positively with the cost of capital in most specifications.

Instead of using the implied cost of capital, we next employ realized returns as the proxy for discount rates and test their connection to risk exposure beliefs in the cross section. This specification resembles that of prior studies on consumption-based asset pricing, including Bansal et al. (2005) and Da (2009). Panel B of Table 4 report our regression results. Consistent with our results for the implied cost of capital, realized future returns are also positively associated with risk exposure beliefs. The positive link is statistically significant at the 1% or 5% level in both univariate and multivariate settings, except when the NAICS industry classification is used to identify peers. Economically, a one-standard-deviation increase in $m_{b,t}$ is accompanied by, on average, a 0.71% rise in realized annual returns.

In sum, we reveal that Bayesian beliefs about risk exposure are priced in the cross section of the cost of capital. This finding supports our intuition that parameter beliefs shape firms' discount rates, which in turn simultaneously impact firm policy and valuation.

3.7. Peer learning

A key identification assumption of our Bayesian learning is that firms in the same industry share the same risk exposure parameter b. As a result, it is optimal for an individual firm to include the cross section of productivity growth from its industry peers when it learns about its own risk exposure. In this section, we test whether this assump-

Table 4

Risk Exposure Beliefs and the Cost of Capital. This table presents the Fama-MacBeth regressions of the cost of capital on risk exposure beliefs and controls. The cost of capital is measured by either the implied cost of capital (ICC) from accounting information (Panel A) or realized return (Panel B). ICC and all regressors are measured at the beginning of each year and realized returns from July to June of the following year. Mean risk exposure beliefs $m_{b,t}$ are calculated from cross-sectional observations of industry constituents, which are identified based on SIC or NAICS codes or the text-based classification system TNIC. The controls are the log market capitalization (ME), investment rate (I/K), and profitability. All regressors are standardized. t-statistics are presented in parentheses below the parameter estimates. *, **, and *** denote statistical significance at the 10%, 5%, and 1% levels, respectively.

Specification Industry class.	(1) SIC	(2) NAICS	(3) TNIC	(4) SIC	(5) NAICS	(6) TNIC
Panel A: Implied Cost of Capital						
$m_{b,t}$	0.0035**	0.0039***	0.0060***	0.0029***	0.0011	0.0045**
	(2.14)	(2.68)	(2.80)	(2.72)	(0.90)	(2.17)
ME				-0.0490***	-0.0484***	-0.0323***
				(-7.36)	(-7.43)	(-9.43)
I/K				-0.0239***	-0.0259***	-0.0229***
				(-7.73)	(-8.54)	(-7.66)
Profitability				-0.0300***	-0.0276***	-0.0279***
				(-4.80)	(-5.18)	(-5.59)
N	91,486	86,364	61,292	91,486	86,364	61,292
adj. R^2	0.00005	0.0001	0.0009	0.036	0.033	0.016
Panel B: Realized Returns						
$m_{b,t}$	0.0072**	0.0036	0.0104**	0.0047**	-0.0002	0.0049***
	(2.01)	(0.73)	(2.61)	(2.35)	(-0.05)	(5.63)
ME				-0.0112	-0.0197*	-0.0198
				(-0.93)	(-1.86)	(-1.38)
I/K				-0.0294***	-0.0291***	-0.0269**
				(-3.58)	(-3.87)	(-2.47)
Profitability				0.0361***	0.0349***	0.0238*
				(4.28)	(5.25)	(1.88)
N	108,349	103,425	73,411	108,349	103,425	73,411
adj. R^2	0.00009	0.00008	-0.0007	0.0053	0.0054	0.0034

tion holds in the data. We do so by using three different approaches. First, we show that individual learning displays insignificant results in explaining firm variables. Second, we test whether the industry classification systems are informative when compared to a random industry assignment. Third, we measure the spillover effect from learning on firm variables when firms learn only from their peers, hence ignoring their own history. In sum, all three experiments support the assumption that industry peers share the same risk exposure parameter.

3.7.1. Learning from firms' individual history

Compared to our benchmark results in which firms learn from peers, we consider here an alternative form of Bayesian learning in which each firm uses only its own history of productivity growth as a signal. This alternative form is worth considering because the classification systems, which we employ to identify industries, could be too noisy (e.g., Bhojraj et al. (2003)). In fact, even firms in the same industry might have very different business profiles, such that peers' observations might not accurately reflect each other's risk profiles. If this is indeed the case, focusing instead on a firm's individual history would result in more precise estimates. To entertain this possibility, we test whether the alternative estimates of beliefs $m^i_{b,t}$ derived from individual learning predict firm variables.

In Table 5, specifications (1) through (3) show regressions when individual learning is employed. We find that the beliefs from individual history are only weakly connected to firm outcomes, and the alternative beliefs relate to firm investment at only the 10% significance. Furthermore, their links to Q and the implied cost of capital are statistically insignificant. This poor performance contrasts with our baseline findings that beliefs based on peer learning are a robust predictor of the same variables.

Why do firms respond only weakly to risk exposure estimates based on their own history? The main difference between the two forms of learning rests primarily in the number of observations involved. Our benchmark estimation of beliefs includes peer observations as an information source, which offers decision makers much richer data from which to learn than does individual learning.

In this particular context of learning about risk exposure, the distinction in the number of observations leads to noticeably different empirical results. The primary source of information is realized productivity growth, which contains substantial noise. In the structural estimation of our model in Section 4, the volatility of idiosyncratic productivity shocks (noise) is 92% annually relative to 2% volatility of consumption shocks (signal). Given this remarkably low signal-to-noise ratio, reliable identification of the risk exposure parameter requires a fairly large number of observations. To that end, individual learning lacks sufficient observations and thus leads to inaccurate risk estimates, which are unable to explain firm outcomes.

3.7.2. Counterfactual industry assignments

In our previous analysis, we showed that peer learning dominates individual learning in explaining firm variables. We next examine the extent to which these industry classification systems help to identify informative signals in the cross section. If the risk exposure is similar across firms irrespective of industry, then the histories of any group of firms would be informative with respect to this parameter. In this case, industry codes could not identify particularly informative signals in the cross section.

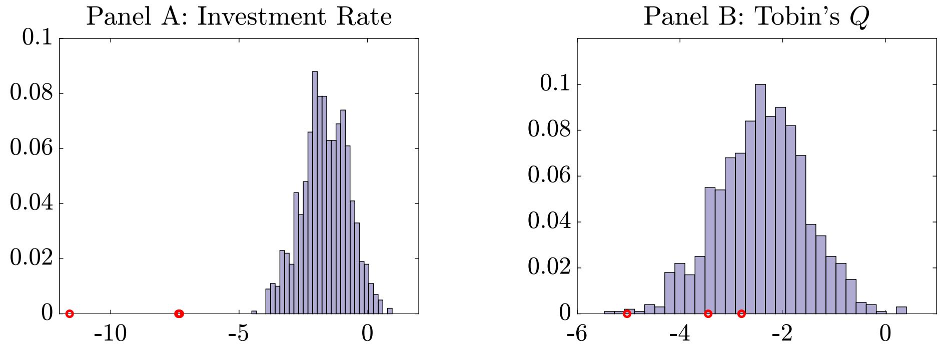

Fig. 4. Learning about Risk Exposure from Counterfactual Industry Peers. This figure plots the histograms of t-statistics of the regression slope coefficients, based on 1,000 counterfactual experiments. We regress investment rates (Panel A) and the Q (Panel B) on risk exposure beliefs and control variables as in regression models (7) and (8), respectively. Beliefs about each firm's risk exposure are updated from observations of counterfactual industry peers, which are randomly assigned. The dots on the x-axis indicate the t-statistics of our baseline results, which are based on actual SIC, NAICS, and TNIC industry classifications.

Motivated by this conjecture, we consider learning from random groupings of firms and assess the resulting risk estimates' predictive power for firm variables in comparison to learning from the actual systems of industry classification (SIC, NAICS, and TNIC). The test is designed as follows. First, we create 385 hypothetical industries to match the total number of industries in the SIC system. Second, each firm is randomly assigned to an industry with five constituents (i.e., the average number of firms for SIC industries). Once assigned, each firm's industry assignment is fixed over time. In turn, firms use the past observations of their counterfactual peers and their own history to update beliefs about their respective risk exposure.

Using the risk exposure beliefs estimated from counterfactual peers, we conduct regressions (7) and (8) to test the responses of firm investment and valuation. We repeat this experiment 1,000 times and present the histograms of t-statistics for the slope coefficient on $m_{b,t}$ in Fig. 4.

If the industry classifications were irrelevant, then the risk exposure beliefs estimated from these counterfactual peers would strongly predict firm variables, as is the case with our baseline estimate. However, we find that the predictive power derived from random peers is noticeably lower than that of the baseline estimates. In the investment regression, in which the risk exposure beliefs should negatively predict investment, the estimates from every actual classification–derived from SIC, NAICS, or TNIC–far outperform all counterfactual estimates, as displayed by the absolute magnitude of t-statistics.

The actual industry classifications continue to play a significant role in predicting Q. The t-statistics of the risk estimates from the actual classification are greater in absolute magnitude than 685 out of 1,000 counterfactual estimates. We note that each firm's own observations are still incorporated in these counterfactual estimates. Hence, the above distinction in the predictive power all depends upon whether or not the parameter learning refers to actual peers. That said, industry classifications are critical to identifying peers that provide informative signals.

3.7.3. Spillover in firm variables

The cross-sectional learning that we propose induces an interdependence among firms: each firm's decisions are affected by its peers' observations. To highlight this relationship, we examine whether firm variables exhibit spillover effects. The basic idea is to consider an alternative estimation of risk exposure beliefs based solely on peer observations without reference to each firm's own history, denoted by $m_{b,t}^{-i}$. We expect that these alternative beliefs will predict firm i's decisions if agents do refer to industry peers in their parameter learning. To test this hypothesis, we conduct regressions (7), (8), and (9), replacing $m_{b,t}$ with $m_{b,t}^{-i}$.

Table 5 presents our regression results in specifications (4) to (6). We confirm the spillover effect in all firm variables. $m_{b,t}^{-i}$ is a strong negative predictor of investment and the total Q. The results indicate that when peers' productivity shocks imply a greater risk exposure, firms in the same industry see their market values decline and reduce their investment. Consistently, the implied cost of capital is strongly positively connected to their peer-inspired beliefs $m_{b,t}^{-i}$. All of these findings support our hypothesis that peers' observations are used to form parameter beliefs.

Compared to our baseline estimation, which utilizes firms' own observations as well as those of their peers, firm responses to peer-inspired beliefs are relatively weaker, as indicated by regression coefficients that are slightly lower in absolute magnitude. This difference implies that firms' own observations constitute an important source of learning. However, firms' individual histories alone do not suffice in this learning context as we document in Section 3.7.1, and their information is optimally used only in conjunction with their peers' histories.

4. Model

The goal of this section is to provide an economic model that explains the empirical link between firms' beliefs about their consumption risk exposure and their investment decisions. To this end, we solve a neoclassical investment model in which firms learn about their productivity exposure to aggregate risk over time. In this model, firms observe their own productivity growth and aggregate consumption growth as signals. Moreover, firms observe the productivity growth of their peers in the same industry. This set of signals is informative because firms in the same industry share the same exposure to consumption risk.

4.1. Stochastic discount factor

For the sake of tractability, we do not model a full general equilibrium model; instead, we specify an exogenous pricing kernel as in Berk et al. (1999). While the majority of the production-based asset pricing literature specifies the pricing kernel as a function of aggregate productivity shocks, we model it as a function of aggregate consumption shocks, similar to Kuehn and Schmid (2014). As such, this paper attempts to bridge the gap between the production-based and consumption-based asset pricing literature.

Specifically, we assume that the log stochastic discount factor is given by

$$\log M_{t+1} = -r_f - \gamma \eta_{t+1} - 0.5\gamma^2, \tag{10}$$

where r_f denotes the log risk-free rate, γ the price of consumption risk, and η_{t+1} is the aggregate shock to consumption growth as in equation (1). For the sake of simplicity, this pricing kernel does not feature time variation in its conditional moments, such as the risk-free rate or price

Table 5
Individual Learning and Spillover Effects from Peers. This table presents regressions of firm variables on alternative measures of risk exposure beliefs and controls. $m^{i}_{b,t}$ is estimated from each firm's own observations only (individual learning), and $m^{-i}_{b,t}$ is based on peer observations only excluding a firm's own observations (peer-only learning). Industry peers are identified by their SIC code. Specifications (1), (2), (4), and (5) present panel regressions of either investment rates ($I_{i,t}/K_{i,t}$) or $Q_{i,t}$. The controls are size, age, profitability, and leverage. Industry fixed effects are also included, and standard errors are clustered by firms. Specifications (3) and (6) present Fama-MacBeth regressions of the implied cost of capital ($\text{ICC}_{i,t}$). t-statistics are presented in parentheses below the parameter estimates. *, **, and *** denote statistical significance at the 10%, 5%, and 1% levels, respectively.

	Individual learning			Peer-only learning		
Specification	(1)	(2)	(3)	(4)	(5)	(6)
Dep. variable	$I_{i,t}/K_{i,t}$	$Q_{i,t}$	$\text{ICC}_{i,t}$	$I_{i,t}/K_{i,t}$	$Q_{i,t}$	$\text{ICC}_{i,t}$
$m^{i}_{b,t}$	-0.0001*	-0.00002	0.2940			
	(-1.71)	(-1.30)	(1.00)			
$m^{-i}_{b,t}$				-0.0021***	-0.0162***	0.0031*
				(-4.55)	(-2.84)	(1.85)
Controls	Yes	Yes	No	Yes	Yes	No
Industry FE	Yes	Yes	n.a.	Yes	Yes	n.a.
Year FE	No	No	n.a.	No	No	n.a.
N	111,259	120,441	90,680	111,259	120,441	90,680
adj. R^2	0.116	0.129	-0.017	0.116	0.129	0.00005

of risk. A similar specification can be found in Carlson et al. (2004) and Hackbarth and Johnson (2015).[21]

4.2. Firms' problem

As in Section 2, firms generate output $Y_{i,t}$ according to a decreasing returns to scale production technology that uses capital as input, as specified in equation (5). Firm-level productivity growth $g_{i,t+1}$ is a mixture of an idiosyncratic and aggregate shock and given by equation (2). Firms do not know their productivity exposure to consumption risk b and learn about this parameter from the cross section of productivity growth of firms within their industry. Since the Bayesian posterior has a normal distribution, the mean and variance of beliefs follow a recursive structure, as specified in equations (3) and (4), respectively.

The capital stock $K_{i,t}$ accumulates according to

$$K_{i,t+1} = (1-\delta)K_{i,t} + I_{i,t}, \tag{11}$$

where $I_{i,t}$ denotes investment and δ the depreciation rate. As is common in the literature, we assume that firms face convex adjustment costs, given by $\phi/2\left(I_{i,t}/K_{i,t}\right)^2 K_{i,t}$, in which ϕ denotes the adjustment cost parameter.

Firms' net payouts $D_{i,t}$ equal output net of investment and adjustment costs and are given by

$$D_{i,t} = X_{i,t}^{1-\alpha} K_{i,t}^{\alpha} - I_{i,t} - \frac{\phi}{2}\left(\frac{I_{i,t}}{K_{i,t}}\right)^2 K_{i,t}. \tag{12}$$

Firms are all equity financed, exit the economy with probability π, and choose investment to maximize their market value, as given by

$$V(S_{i,t}) = \max_{I_{i,t}} \left\{ D_{i,t} + (1-\pi)\tilde{\mathbb{E}}_t\left[M_{t+1}V(S_{i,t+1})\right]\right\}, \tag{13}$$

where $S_{i,t} = (X_{i,t}, K_{i,t}, m_{b,t}, \sigma_{b,t})$ is the vector of state variables. In the case of exit, the capital stock of the incumbent becomes worthless because the technology of the entrant renders the old technology obsolescent. The tilde above the expectation operator means that the integration over future shocks is conducted under the agent's subjective beliefs. In addition to productivity and capital, Bayesian learning about the risk exposure parameter implies that the mean and variance of the posterior distribution are state variables, whose dynamics are given by equations (3) and (4), respectively.

A few distinctive features of this firm's problem are noteworthy. First, the optimal investment policy is a function of the agent's beliefs (i.e., $I_{i,t} = I(S_{i,t})$), which implies that revisions in beliefs affect real outcomes. Second, firm i's investment policy and valuation ratios are influenced by its peers' histories of productivity shocks, via the updating of beliefs. Both of these model features are supported by the data.

Given a firm's value, we can compute returns $R_{i,t+1} = V_{i,t+1}/(V_{i,t} - D_{i,t})$ and excess returns $r_{i,t+1} = R_{i,t+1} - r_f$. We solve the firm's problem numerically and obtain the firm's investment rate $I_{i,t}/K_{i,t}$, Tobin's Q ratio $Q_{i,t} = (V_{i,t} - D_{i,t})/K_{i,t}$, and expected returns $\tilde{\mathbb{E}}_t[R_{i,t+1}]$ as a function of the state variables. We provide additional details about the numerical solution in Appendix B.

4.3. Analytical solutions

Before solving the firm's problem numerically, we consider a simplified model version in which the risk exposure parameter is known and capital adjustment is frictionless. This simplification lets us obtain analytic expressions for the investment policy of firms, market values, and expected stock returns. Using explicit solutions, we analyze the dependence of these firm variables on the risk exposure parameter.

The following lemma presents the optimal investment policy and the valuation ratio.

Lemma 1. *When the risk exposure parameter is known and capital adjustment is frictionless, the optimal capital policy solving the value function (13) is given by*

$$K_{i,t+1} = \tau^* X_{i,t} \qquad \tau^* = \chi e^{0.5\left[(1-\alpha)b\sigma_c - \gamma\right]^2/(1-\alpha)} \tag{14}$$

and average Q is

$$Q = \frac{\kappa_1}{1-\kappa_2 e^{0.5(b\sigma_c-\gamma)^2}} + 1, \tag{15}$$

where χ, κ_1, and κ_2 are all positive constants.

[21] This pricing kernel could be motivated with Epstein-Zin preferences, when consumption growth follows an i.i.d. normal process. While risk preferences also affect the risk-free rate under power utility, Epstein-Zin preferences allow for separate risk and time preferences.

Table 6
Sensitivity Matrix. This table shows the sensitivity of model-implied moments (in rows) with respect to model parameters (in columns). The sensitivity of moment i with respect to parameter j equals $\frac{\partial g_i^M}{\partial \theta_j}\frac{\theta_j}{g_i^M}$ and is evaluated at the vector of point estimates from Table 7.

		δ	α	ϕ	σ	b	γ
Investment rate	*mean*	3.13	-1.52	0.01	-1.65	-0.03	-0.10
	var.	1.57	-8.05	-1.34	0.61	-0.01	-0.05
Stock return	*mean*	0.36	-4.71	-0.04	-0.67	0.30	0.86
	var.	1.06	-12.75	-0.38	1.06	-0.10	-0.10
Tobin's Q	*mean*	1.67	-7.79	0.63	-0.96	-0.13	-0.21
	var.	3.80	-25.96	1.20	1.41	-0.56	-0.79
Bayesian posterior mean	*mean*	0.00	0.00	0.00	-0.45	0.42	0.00
	var.	0.00	0.00	0.00	1.30	0.13	0.00

The proof of this lemma is provided in Appendix C. Without frictions in capital adjustment, firms choose a constant capital-productivity ratio τ^*. Due to the i.i.d. stochastic discount factor and constant risk exposure, the discount rate is static, rendering average Q constant. Using these expressions, we can easily show that $\partial\tau^*/\partial b < 0$ for $b < \gamma/\left[(1-\alpha)\sigma_c\right]$, and $\partial Q/\partial b < 0$ for $b < \gamma/\sigma_c$.[22] Intuitively, under realistic values of b, greater risk exposure of productivity causes the present value of the marginal product of capital to fall. As a result, the optimal capital policy weakens and thus investment decrease, as does the valuation ratio.

The firms' expected return is presented in the next lemma.

Lemma 2. *When the risk exposure parameter is known and capital adjustment is frictionless, the expected gross return is a constant given by*

$$\mathbb{E}\left[R_{i,t}\right] = (1-\pi)\frac{1-\kappa_2 e^{0.5(b\sigma_c-\gamma)^2}}{\kappa_1 + 1 - \kappa_2 e^{0.5(b\sigma_c-\gamma)^2}} \times \left[\kappa_3 e^{r_f+(1-\alpha)b\sigma_c\gamma} + (1-\delta) + \frac{\kappa_1 e^{\mu+0.5b\sigma_c^2+0.5\sigma^2}}{1-\kappa_2 e^{0.5(b\sigma_c-\gamma)^2}}\right]. \quad (16)$$

In Appendix C, we prove that $\partial\mathbb{E}\left[R_{i,t}\right]/\partial b$ is positive under specific conditions on b, which hold for realistic parameter values. Thus, this analytic model explicitly states a positive link between risk exposure and expected returns.

Derived from the simplified setup, these predictions are qualitatively consistent with our empirical results. In the following, we will show that the same results also hold quantitatively in the full model, which features capital adjustment costs and learning.

4.4. Estimation

To quantitatively evaluate the model performance, we calibrate some parameters that can be easily measured in the data and structurally estimate the remaining parameters, for which the existing literature provides only weak priors. In particular, we calibrate the consumption process and risk-free rate to the data in Beeler and Campbell (2012). They report that consumption growth has an average growth rate μ of 1.93% and a volatility σ_c of 2.16% over the long sample of 1930-2008. The average risk-free rate r_f is 0.56%. These parameters are relevant for the pricing kernel.

The remaining model parameters are estimated with the simulated method of moments (SMM). Given the predefined parameters, we estimate the depreciation rate δ, capital share of production α, adjustment cost parameter ϕ, idiosyncratic volatility σ, and productivity exposure to consumption risk b on the firm side, as well as the price of risk γ in the pricing kernel. For a given parameter vector $\theta = (\delta, \alpha, \phi, \sigma, b, \gamma)$, we solve the model numerically at an annual frequency.

We simulate 1,000 economies, each consisting of 385 industries for 57 years with 5 years of burn-in. Initially, each industry has 5 peer firms, which represents the industry average. Firms exit the economy with a probability of 1/14 to mimic the average firm lifespan of 14 years. When firms exit, new firms enter the economy, and they randomly draw their industry classification from a discrete uniform distribution with support between 1 and 385. As a result, the number of industry peers varies over time as in the data. We set the initial prior beliefs to have a mean $m_{b,0}$ of one and a dispersion $\sigma_{b,0}$ of five, as in the empirical exercise.

Based on the simulated data, we calculate the model moments. The SMM objective function J equals a weighted distance metric between moments from actual data g_D and model moments g_M with weighting W, i.e., $J(\theta) = [g_D - g_M(\theta)]^\top W[g_D - g_M(\theta)]$. The efficient weighting matrix is the inverse of the sample covariance matrix of the moments, which we estimate using influence functions clustered at the industry level, as in Hennessy and Whited (2007). Since covariances are estimated with considerable noise, we use only its diagonal elements to compute the weighting matrix, similar to Schroth et al. (2014). We obtain the parameter estimate $\hat{\theta}$ by searching globally over the parameter space, which we implement via a particle swarm algorithm.

We identify the six parameters $(\delta, \alpha, \phi, \sigma, b, \gamma)$ based on eight moments. In the estimation, we include the mean and variance of the investment rate $I_{i,t}/K_{i,t}$, equity excess returns $r_{i,t}$, Tobin's $Q_{i,t}$, and the posterior mean belief $m_{b,t}$. The variances are demeaned at the industry level, as in the data, to remove persistent differences across industries, for which the model cannot account. While each parameter affects multiple moments, it is useful to discuss the main sources of identification. Table 6 reports the sensitivity of moment i with respect to parameter j, $\frac{\partial g_i^M}{\partial \theta_j}\frac{\theta_j}{g_i^M}$, evaluated at the point estimates from Table 7.

The depreciation rate is identified by the average investment rate because firms must invest more when capital depreciates at a faster rate. The capital share of production is related to the mean and variance of returns and Tobin's Q. As the capital share rises, output growth becomes less volatile, as reflected in equation (5). This effect reduces the volatility of dividend growth. As a result, returns are less volatile, and expected returns are lower.

A rise in the capital share also lowers the growth rate of productivity, which is $(1-\alpha)\mu$. Although lower discount rates lead to an increase in the valuation ratio Q, lower growth rates cause this valuation ratio to drop; here, the second effect dominates. Capital adjustment costs are pinned down by the variance of the investment rate. As capital adjustments become more costly, it is optimal for firms to make small investments more frequently.

Idiosyncratic risk positively impacts the variance of investment rates, returns, Tobin's Q, and average Bayesian beliefs. The risk exposure parameter is related to average returns and beliefs. As firms are

[22] According to our estimation, $\gamma/\sigma_c \approx 109$, and $\gamma/\left[\sigma_c(1-\alpha)\right] \approx 431$. Risk exposure affects the dispersion in productivity and also the expected growth rate due to a Jensen effect. If b exceeds these thresholds, then the growth rate effect dominates the discount rate effect.

Table 7
SMM Estimation. This table summarizes the SMM estimation of six model parameters: depreciation rate δ, capital share of production α, adjustment cost parameter ϕ, volatility of idiosyncratic productivity σ, exposure to consumption risk b, and the price of risk γ. The estimation targets the mean and variance of investment rates, Tobin's Q, stock returns, and the Bayesian posterior mean risk exposure. Standard errors are reported in parentheses and based on the sample covariance matrix of the moments, which we estimate using influence functions clustered at the industry level.

Panel A: Parameter Estimates

Parameter		Estimates
Depreciation rate	δ	0.5570
		(0.0046)
Capital share of production	α	0.7481
		(0.0027)
Adjustment cost parameter	ϕ	2.1541
		(0.0651)
Volatility of idiosyncratic productivity	σ	0.9206
		(0.0094)
Exposure to consumption risk	b	5.0107
		(0.0752)
Price of risk	γ	2.3481
		(0.0116)

Panel B: Moments

Moments		Data	Model
Investment rate	*mean*	0.2466	0.2457
	s.d.	0.2113	0.2203
Stock return	*mean*	0.0987	0.0993
	s.d.	0.5757	0.5314
Tobin's Q	*mean*	1.9399	2.0173
	s.d.	2.2557	2.0722
Bayesian posterior mean	*mean*	3.6217	3.5476
	s.d.	1.8068	1.8887

more exposed to aggregate consumption risk, risk premia rise. As the true risk exposure parameter increases, the average Bayesian belief does so as well. The price of risk is identified by average returns because this parameter increases the curvature of the pricing kernel.

4.5. *Estimation results*

Table 7 summarizes the point estimates and moment conditions. Overall, the model matches all moments well. Both in the model and data, investment rates average around 25% annually with a standard deviation of around 21%. In the data, stock returns are close to 3 times more volatile than investment rates, which the model can almost replicate. Specifically, stock returns have a volatility of 58% in the data, compared to 53% in the model. The model can match these moments with a large depreciation rate of 56%, an idiosyncratic volatility of 92%, and an adjustment cost parameter of 2.15. The large adjustment costs drive a wedge between the volatility of investment and stock returns and imply that firms spend around 6.4% of their output on capital adjustments. The model can also match firm-level average excess returns of 10% with a price of risk of 2.35.[23]

The model also replicates fairly well an average Q of around 1.9 with a capital share of 0.75. Relative to the existing literature (e.g., Nikolov and Whited (2014)), our model also generates a volatile Tobin's Q. Intuitively, the time variation of valuation ratios such as Q reflects time variation in discount rates. Learning about risk exposure generates time variation in discount rates, even though the true parameter is constant. To understand the link between learning and uncertainty, we plot in Fig. 5 the volatility of stock returns (Panel A) and Tobin's Q (Panel B) as a function of the number of peers n, when the risk exposure b is subject to learning, denoted by plus signs. The blue dashed line shows the volatility in case b is known, and the red solid line represents the data moment.

When the number of peers increases, the volatility of stock returns and Tobin's Q decreases because the agent observes more signals from which to learn. However, even with ten industry peers, our learning model significantly improves on explaining the magnitude of the volatility of stock returns and Q compared to the literature, which has ignored parameter uncertainty. In our benchmark specification, each industry is assumed to have five firms to reflect the average number in the data. Stock returns and Q are not as volatile as in the data but the investment rate is slightly too volatile, which restricts the estimation to increase idiosyncratic risk. With fewer industry peers, the model can match either the high volatility of stock returns or Tobin's Q.

Lastly, the estimated risk exposure parameter is 5.0, even though the average risk exposure is only 3.6. The reason for this wedge is that we assume a date-0 prior for the mean risk exposure of one in the model and data. This estimate implies that 12% of the standard deviation of firm-level productivity growth comes from aggregate risk. As a comparison, Bansal and Yaron (2004) assume a volatility scaling parameter of 4.5 for aggregate dividends.

4.6. *Implications*

In this section, we compare model implications with the data that were not targeted by the estimation, and can thus be viewed as an out-of-sample validation of our model.

Firstly, our model replicates the productivity commonality among industry peers. To illustrate this point, we compare in Fig. 6 the percentage of productivity variance explained by the first principal component between data and model. Panel A displays the histogram of the explained variance pooled across 10-year subperiods from Fig. 2. Panel B shows the histogram from model simulations.[24] The empirical distribution has more variability than the theoretical one; that said, the averages are fairly close. In particular, the average percentage of variance explained by common fluctuation is 36.2% in the model, compared to 35.9% in the data. This match provides empirical support for our productivity process.

We next explore the model implications for how risk exposure beliefs should affect firm decisions. Fig. 7 depicts the investment rate I_t/K_t, Tobin's Q, and expected returns as a function of the mean risk exposure $m_{b,t}$, evaluated at the average capital stock. Expected returns increase in the average risk exposure because a large fraction of the volatility of productivity arises from systematic risk. As discount rates rise in average risk exposure, they depress both investment and the Q valuation ratio. These relationships are consistent with the theoretical predictions we derived from the simplified model in Section 4.3.

Next, we examine the quantitative association between risk exposure beliefs and firm variables in the simulated model. To this end, we replicate regressions (7), (8), and (9) on the 1,000 economies that we simulated for the estimation. In the model, we measure size as the logarithm of the capital stock, profitability as dividends divided by capital, and market equity (ME) as the logarithm of firm value.

In Table 8, we report the cross-simulation averages of regression coefficients. Overall, the model generates strong firm responses to risk exposure beliefs, which are similar to the empirical patterns. The model-implied link between investment and $m_{b,t}$ is negative. The slope coefficient of $m_{b,t}$ is −0.013, implying that a one-standard-deviation increase in $m_{b,t}$ leads to a 10.1% decrease in the investment rate. The magnitude is comparable to the empirical response of 5.6%.

[23] Even though the economic fit of the model is very good, statistically the model is rejected with a J-value of 172.

[24] We generate 100,000 economies, each of which has eight peer firms and lasts for ten years.

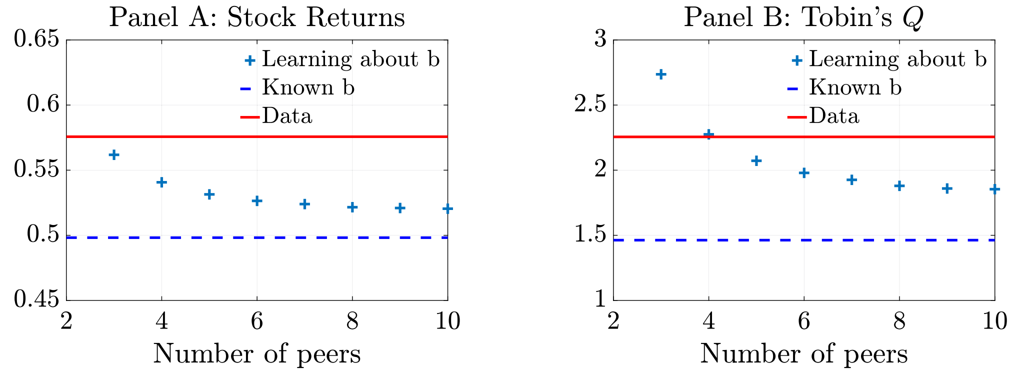

Fig. 5. Volatility of Stock Returns and Tobin's Q. This figure depicts the volatility of stock returns (Panel A) and Tobin's Q (Panel B) as a function of the number of peers, when the risk exposure b is subject to learning, denoted by plus signs. The blue dashed line shows the volatility in case b is known and the red solid line represents the data moment. (For interpretation of the colors in the figure(s), the reader is referred to the web version of this article.)

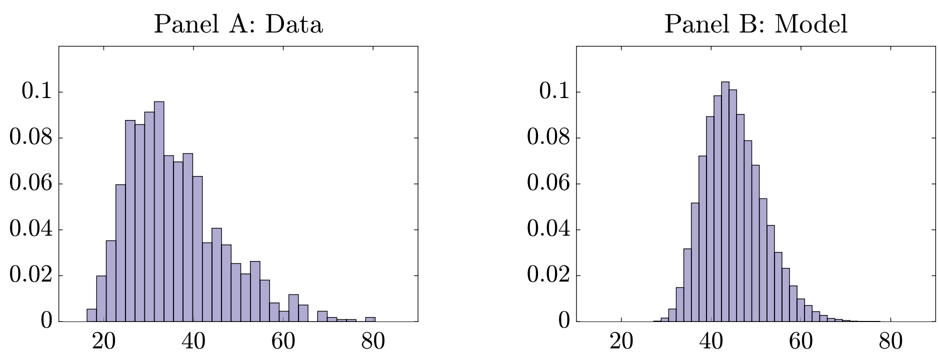

Fig. 6. Common Fluctuations in Productivity. This figure shows the histogram of the explained variance (in percent) generated by the common fluctuation of productivity among industry peers in the pooled data (Panel A) and model (Panel B). The common fluctuation for each industry is identified by the first principal component using subsamples of 10 years of data.

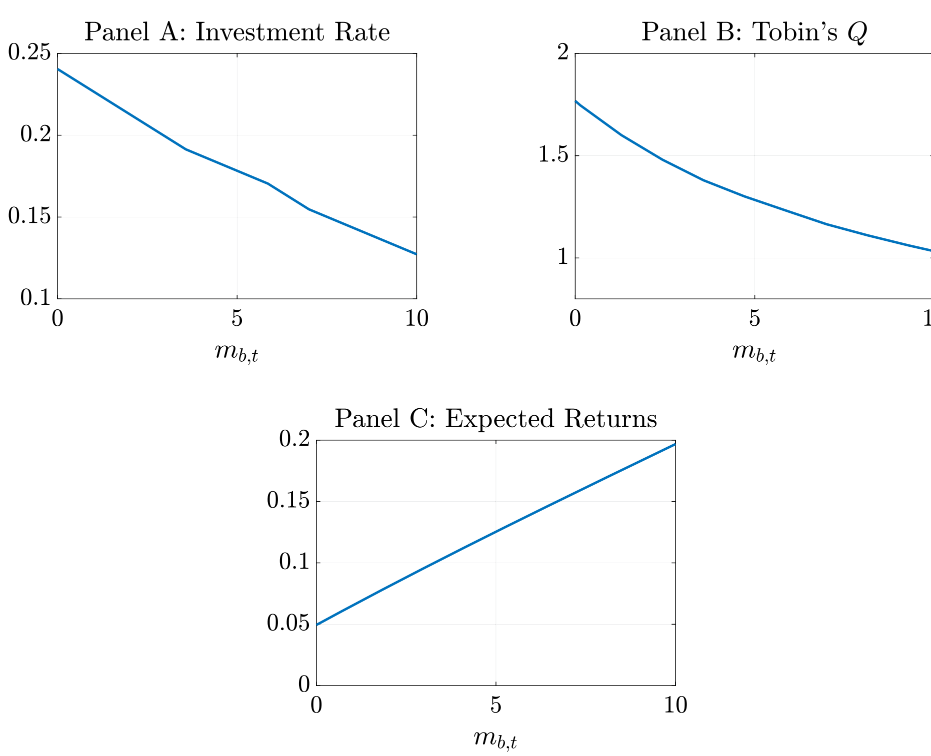

Fig. 7. Risk Exposure Beliefs and Firm Variables. This figure depicts the investment rate I/K, Tobin's Q, and expected excess returns as a function of the mean risk exposure $m_{b,t}$, evaluated at the average capital stock.

Table 8

Risk Exposure Beliefs and Firm Variables in the Model. This table presents panel regressions of firm variables on risk exposure beliefs on simulated data. We simulate 1,000 economies, each containing 385 industries with five peer firms per industry, for 57 years with five years of burn-in, as in the data. The table reports cross-simulation averages of regression coefficients and adjusted R^2. Across specifications, we regress investment rates $I_{i,t}/K_{i,t}$, Tobin's $Q_{i,t}$, expected excess returns $\mathbb{E}_t[r_{i,t+1}]$, or realized excess returns $r_{i,t+1}$ on risk exposure beliefs $m_{b,t}$ and controls. In parentheses, we report the cross-simulation standard deviation of slope coefficients.

Specification Dep. variable	(1) $I_{i,t}/K_{i,t}$	(2) $Q_{i,t}$	(3) $\mathbb{E}_t[r_{i,t+1}]$	(4) $\mathbb{E}_t[r_{i,t+1}]$	(5) $\mathbb{E}_t[r_{i,t+1}]$	(6) $r_{i,t+1}$	(7) $r_{i,t+1}$	(8) $r_{i,t+1}$
$m_{b,t}$	-0.0131 (0.0008)	-0.1512 (0.0341)	0.0360 (0.0015)		0.0307 (0.0014)	0.0346 (0.0022)		0.0290 (0.0036)
Size	-0.0216 (0.0042)	1.1347 (0.2383)						
Profitability	0.4013 (0.0221)	6.8173 (1.4024)		0.1719 (0.0110)	0.0357 (0.0042)		0.1636 (0.0211)	0.0351 (0.0260)
ME				-0.0054 (0.0027)	-0.0015 (0.0025)		-0.0062 (0.0062)	-0.0026 (0.0062)
I/K				-0.1698 (0.0090)	-0.0310 (0.0041)		-0.1615 (0.0176)	-0.0306 (0.0237)
adj. R^2	0.973	0.787	0.944	0.772	0.986	0.004	0.004	0.005

Table 9

Spillovers in Firm Variables in the Model. This table presents panel regressions of firm variables on risk exposure beliefs on simulated data. We simulate 1,000 economies, each containing 385 industries with five peer firms per industry, for 57 years with five years of burn-in, as in the data. The table reports cross-simulation averages of regression coefficients and adjusted R^2. Across specifications, we regress investment rates $I_{i,t}/K_{i,t}$, Tobin's $Q_{i,t}$, expected excess returns $\mathbb{E}_t[r_{i,t+1}]$, or realized excess returns $r_{i,t+1}$ on the posterior mean belief $m_{b,t}^{-i}$, which is based on peer observations only, excluding each firm's own history, and controls. In parentheses, we report the cross-simulation standard deviation of slope coefficients.

Specification Dep. variable	(1) $I_{i,t}/K_{i,t}$	(2) $Q_{i,t}$	(3) $\mathbb{E}_t[r_{i,t+1}]$	(4) $\mathbb{E}_t[r_{i,t+1}]$	(5) $\mathbb{E}_t[r_{i,t+1}]$	(6) $r_{i,t+1}$	(7) $r_{i,t+1}$	(8) $r_{i,t+1}$
$m_{b,t}^{-i}$	-0.0056 (0.0014)	-0.0678 (0.0217)	0.0158 (0.0011)		0.0045 (0.0006)	0.0154 (0.0020)		0.0045 (0.0019)
Size	-0.0221 (0.0040)	1.1297 (0.2413)						
Profitability	0.3990 (0.0221)	6.7912 (1.3987)		0.1719 (0.0110)	0.1627 (0.0110)		0.1636 (0.0211)	0.1545 (0.0216)
ME				-0.0054 (0.0027)	-0.0051 (0.0026)		-0.0062 (0.0062)	-0.0060 (0.0062)
I/K				-0.1698 (0.0090)	-0.1605 (0.0092)		-0.1615 (0.0176)	-0.1523 (0.0183)
adj. R^2	0.965	0.777	0.184	0.772	0.785	0.0009	0.004	0.004

Furthermore, the model produces a positive relationship between risk exposure beliefs and the cost of capital, which resembles the regularity in the data. On simulated data, both expected excess returns and future realized excess returns increase with the posterior mean risk exposure. In univariate regressions presented by specifications (3) and (6), a one-standard-deviation increase in $m_{b,t}$ raises the expected return (realized return) by 3.60% (3.46%) per year. These model responses are reasonably close to the empirical reaction. For the same change in $m_{b,t}$, the ICC (realized return) moves upward by 0.45% (0.71%) on average in the data.

Since we assume the existence of a unique pricing kernel (10), the fundamental equation of asset pricing, $\mathbb{E}_t[M_{t+1}(1 + R_{i,t+1})] = 1$, implies that excess returns are given by $\mathbb{E}_t[r_{i,t+1}] = \beta_{i,t}\lambda$, where $\beta_{i,t}$ denotes the conditional beta between the pricing kernel and returns and λ the price of risk. Importantly, the cash flow beta $m_{b,t}$ is different from the return beta, which would align perfectly with expected returns in equilibrium; nevertheless, the cash flow beta alone explains 94% of the cross-sectional variation in expected returns, as indicated by the R^2 in specification (3). Interestingly, the cash flow beta dominates characteristics in specification (4), in which we control for profitability, market equity, and investment rates. Similar results hold for realized returns in specifications (6) to (8).

Since in the model, we can exactly identify expected excess returns, the regression R^2 for the univariate cash flow beta regression exceeds 94%. When we instead use realized excess returns, the regression R^2 drops below 1%, which is in line with the empirical evidence in Table 4. Since realized excess returns equal expected excess returns plus idiosyncratic noise, this finding indicates that our model replicates the correct empirical composition of idiosyncratic to aggregate risk.

In addition, we confirm that our model reproduces spillover effects. In Table 9, we present the model-implied responses to beliefs derived from peer observations. As in the data, $m_{b,t}^{-i}$ negatively predicts both investment and the valuation ratio and, simultaneously, positively the cost of capital. These findings confirm the model's ability to capture that peer observations shape the beliefs about a firm's risk profile. Compared to Table 8, in which both firms' own and peers' observations are used in learning, firms respond relatively weakly to beliefs influenced by peers only. This comparison also aligns with our empirical evidence.

Table 10

Time Variation in Risk Exposure. This table presents panel regressions of firm variables on risk exposure beliefs. Across six specifications, we regress investment rates $I_{i,t}/K_{i,t}$, $Q_{i,t}$, or the implied cost of capital $\text{ICC}_{i,t}$ on beliefs about time-varying risk exposures and controls. The beliefs consist of the posterior mean about the unconditional risk exposure $\bar{b}$ denoted by $m^{\text{KF}}_{\bar{b},t}$ and the posterior mean about the conditional risk exposure b_t denoted by $m^{\text{KF}}_{b_t,t}$, which are estimated using the Kalman filter utilizing the cross-sectional observations of SIC peers. Specifications (1) through (4) present panel regression results with industry fixed effects. Controls are size, age, profitability, and leverage; standard errors are clustered by firms. Specifications (5) and (6) present Fama-MacBeth regression results. *, **, and *** denote statistical significance at the 10%, 5%, and 1% levels, respectively.

Dep. variable	$I_{i,t}/K_{i,t}$		$Q_{i,t}$		$\text{ICC}_{i,t}$	
Specification	(1)	(2)	(3)	(4)	(5)	(6)
$m^{\text{KF}}_{\bar{b},t}$	-0.0273***	-0.0250***	-0.0859***	-0.0385	0.0078***	0.0051**
	(-18.75)	(-14.54)	(-3.62)	(-1.45)	(3.25)	(2.41)
$m^{\text{KF}}_{b_t,t}$		-0.0013**		-0.0268***		0.0025**
		(-2.41)		(-3.35)		(2.59)
Controls	Yes	Yes	Yes	Yes	No	No
Industry FE	Yes	Yes	Yes	Yes	n.a.	n.a.
Year FE	No	No	No	No	n.a.	n.a.
N	109,375	109,375	116,263	116,263	91,486	91,486
adj. R^2	0.120	0.120	0.122	0.123	0.0004	0.0005

5. Robustness

In this section, we examine the robustness of our findings. First, we consider parameter learning when the true risk exposure is dynamic. Second, we show that firms' response to risk exposure beliefs is distinct from learning about the drift of productivity growth.

5.1. Time variation in risk exposure

One of the stylized assumptions in our model is that the true exposure of productivity to consumption shocks is constant over time. Changes in risk exposure beliefs are not driven by shifts in true risk profiles but are induced by the continuous updating of beliefs from growing observations. That said, what if the true exposure itself fluctuates over time, contrary to our assumption? If so, our learning-based risk estimate might misleadingly capture variation in the true exposure, and thus overstate the importance of learning. To address this concern, we extend our model to incorporate possible shifts of true risk exposure and examine the learning impact in this context.

Specifically, we now assume that the true risk exposure follows a first-order autoregressive process

$$b_{t+1} = \varphi b_t + (1-\varphi)\bar{b} + \sigma_b \xi_{t+1}, \tag{17}$$

where ξ_{t+1} is an i.i.d. standard normal innovation, φ denotes the autocorrelation and $\bar{b}$ the long-run average in risk exposure, and σ_b quantifies the magnitude of time variation in risk exposure. This dynamic exposure b_{t+1} enters the law of motion of productivity in equation (2), replacing the constant exposure b. Similar to the baseline model, we consider an agent who observes neither $\bar{b}$ nor b_t and instead infers them from realized productivity.

In Appendix D, we describe the updating of beliefs about risk exposure in this setting. In sum, agents revise their beliefs about the risk exposure vector $(b_t, \bar{b})$ over time by using the Kalman filter. To conduct this filtering, we estimate the parameters in equation (17) using the expectation-maximization algorithm. Based on the parameter estimates, we obtain the posterior means of $\bar{b}$ and b_t conditional on all observations available at time t, and we denote them by $m^{\text{KF}}_{\bar{b},t}$ and $m^{\text{KF}}_{b_t,t}$. Next, we test whether the firm variables respond to these beliefs. In particular, we focus on their responses to the beliefs about the unconditional mean of risk exposure $m^{\text{KF}}_{\bar{b},t}$. We note that the true value of $\bar{b}$ is constant by nature, so any change in the estimate of this parameter is entirely driven by learning.

We report the regression results in Table 10. Interestingly, the results here echo our baseline findings. In specifications (1), (3), and (5), the posterior mean of unconditional risk exposure $m^{\text{KF}}_{\bar{b},t}$ negatively predicts investment and total Q and positively the cost of capital. All of these associations are statistically significant at the 1% level. Furthermore, when we control for the impact of the conditional risk exposure $m^{\text{KF}}_{b_t,t}$ in specifications (2), (4), and (6), $m^{\text{KF}}_{\bar{b},t}$ continues to connect strongly to investment and cost of capital. Therefore, we confirm that belief updates about the unconditional risk influence corporate decisions and market valuations, apart from what is induced by changes in the true risk profile.

In sum, we conclude that the evolution of risk exposure beliefs is an important consideration in practice, irrespective of whether the true risk exposure is static or dynamic.

5.2. Joint learning about productivity drift and risk exposure

Distinct from our focus on risk exposure, many prior studies have considered uncertainty regarding the drift of productivity (e.g., Pastor and Veronesi (2003), Alti (2003), and Andrei et al. (2019)). In this section, we ensure that our results are separate and not misleadingly driven by learning about the mean growth rate.

We begin this analysis with an alternative hypothesis: agents are uncertain about the drift μ of productivity as well as the risk exposure parameter b. We further assume that μ is identical for all industry peers, though different from the average of consumption growth. Hence, peers' cross-sectional observations of productivity are the primary source of forming beliefs about μ, as we assumed in the case of learning about b.

In Appendix E, we represent this joint learning in a state-space model, where the state variable is a vector of unknown parameters $[\mu, b]^\top$. In turn, beliefs about the unobservable state are estimated using the Kalman Filter. As a result, we obtain the posterior mean of drift, $m^{\text{KF}}_{\mu,t}$, and the posterior mean of risk exposure, $m^{\text{KF}}_{b,t}$, conditional on all observations until time t. Having estimated the parameter beliefs under this joint learning, we examine how firm variables respond to these beliefs.

Table 11 reports the regression results. Consistent with the prior literature, we find that investment rates rise and the costs of capital fall with respect to beliefs about the drift $m_{\mu,t}$. More importantly, when we control for beliefs about the drift, beliefs about risk exposure maintain the strong association with all firm variables. Specifically, $m^{\text{KF}}_{b,t}$ negatively predicts investment rates and Q and positively the cost of capital.

Table 11
Joint Learning about Productivity Drift and Risk Exposure. This table presents panel regressions of firm variables on parameter beliefs. Across three specifications, we regress investment rates $I_{i,t}/K_{i,t}$, $Q_{i,t}$, or the implied cost of capital $\text{ICC}_{i,t}$ on beliefs about risk exposure $m^{\text{KF}}_{b,t}$, beliefs about productivity mean growth $m^{\text{KF}}_{\mu,t}$ and controls. Specifications (1) and (2) present panel regression results with industry fixed effects. Controls are size, age, profitability, and leverage; standard errors are clustered by firms. Specification (3) presents Fama-MacBeth regression results. *, **, and *** denote statistical significance at the 10%, 5%, and 1% levels, respectively.

Specification Dep. variable	(1) $I_{i,t}/K_{i,t}$	(2) $Q_{i,t}$	(3) $\text{ICC}_{i,t}$
$m^{\text{KF}}_{b,t}$	-0.0079***	-0.0312***	0.0044***
	(-15.04)	(-3.67)	(3.67)
$m^{\text{KF}}_{\mu,t}$	0.1631***	0.3393	-0.0120***
	(4.95)	(0.78)	(-8.07)
Controls	Yes	Yes	No
Industry FE	Yes	Yes	n.a.
Year FE	No	No	n.a.
N	109,375	116,263	91,486
adj. R^2	0.115	0.122	0.0008

In all specifications, the coefficient estimate on $m_{b,t}$ remains fairly unchanged even after we include $m_{\mu,t}$ as a control.

We, therefore, conclude that firm responses to risk exposure beliefs are clearly distinct from the conventional implications regarding the uncertain drift of productivity. While it is well known that market participants attempt to forecast cash flow growth rates, our novel finding reveals that they also engage in learning about discount rate characteristics, such as the exposure to aggregate consumption risk.

6. Conclusion

In the consumption-based asset pricing paradigm, firms' exposure to consumption risk is a crucial characteristic and thus should impact firm decisions and valuations. Despite its importance, estimations of the risk exposure parameter have been elusive. Firm observables, which are potentially informative about this parameter, are often primarily driven by idiosyncratic news, thus hampering the identification of the systematic component of cash flows. In response to this challenge, prior studies such as Bansal et al. (2005) and Da (2009), which have attempted to measure consumption cash flow betas, have relied on a portfolio-level analysis and static betas. To overcome these limitations, we propose a neoclassical investment model in which agents gradually learn about the parameter through Bayesian updating. In particular, parameter beliefs are updated from firms' and industry peers' comovement between their productivity and consumption growth.

We empirically establish that this parameter learning shapes firms' real decisions and market valuations. As the Bayesian mean risk exposure is continuously revised over time, discount rates respond positively, while the investment rate and Tobin's Q respond negatively. We find that a key source of learning is cross-sectional information from peers. Alternative beliefs about risk exposure, which ignore peer observations, do not predict firm variables. To further support our empirical findings, we use our structurally estimated neoclassical investment model to reproduce the quantitative links between risk exposure beliefs and firm variables. All these findings suggest that consumption risk is an important consideration in practice, and that the evolution of risk exposure beliefs is priced in financial markets.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Appendix A. Derivation of the Bayesian posterior

For the sake of simplicity, we define the demeaned growth in consumption and firm i's productivity as

$$\bar{g}_{c,t} = g_{c,t} - \mu = \sigma_c \eta_t$$

$$\bar{g}_{i,t} = g_{i,t} - \mu = b\sigma_c \eta_t + \sigma \varepsilon_{i,t}.$$

Consider an industry with n constituting firms. Let $\bar{g}_t$ denote the vector consisting of the constituents' growth, $\bar{g}_t = (\bar{g}_{1,t}, \bar{g}_{2,t}, ..., \bar{g}_{n,t})$. According to Bayes' law, the probability of b conditional on all observations until time t is

$$\begin{aligned}\text{Prob}\left(b|\bar{g}_1, ..., \bar{g}_t, \bar{g}_{c,1}, ..., \bar{g}_{c,t}\right) &\propto \text{Prob}\left(b, \bar{g}_t, \bar{g}_{c,t}|\bar{g}_1, ..., \bar{g}_{t-1}, \bar{g}_{c,1}, ..., \bar{g}_{c,t-1}\right)\\ &\propto \text{Prob}\left(\bar{g}_t|b, \bar{g}_{c,t}\right)\\ &\quad \times \text{Prob}\left(b|\bar{g}_1, ..., \bar{g}_{t-1}, \bar{g}_{c,1}, ..., \bar{g}_{c,t-1}\right),\end{aligned}$$

where we use the fact that $\bar{g}_{c,t}$ is independent of past shocks and $\bar{g}_t$ is conditionally independent of past observations, given b and $\bar{g}_{c,t}$.

Suppose that beliefs about b based on observations until time $t-1$ are normally distributed with mean $m_{b,t-1}$ and standard deviation $\sigma_{b,t-1}$. With the arrival of new observations at time t, the Bayesian posterior becomes

$$\begin{aligned}&\text{Prob}\left(b|\bar{g}_1, \bar{g}_2, ..., \bar{g}_t, \bar{g}_{c,1}, \bar{g}_{c,2}, ..., \bar{g}_{c,t}\right)\\ &\propto \prod_{i=1}^{n} \exp\left(-\frac{(\bar{g}_{i,t} - b\bar{g}_{c,t})^2}{2\sigma^2}\right) \times \exp\left(-\frac{(b - m_{b,t-1})^2}{2\sigma^2_{b,t-1}}\right)\\ &= \exp\left(-\frac{\sum_{i=1}^{n}(\bar{g}_{i,t} - b\bar{g}_{c,t})^2}{2\sigma^2}\right) \times \exp\left(-\frac{(b - m_{b,t-1})^2}{2\sigma^2_{b,t-1}}\right)\\ &\propto \exp\left(-\frac{b^2 - 2\frac{\sigma^2_{b,t-1}\sum_{i=1}^{n}\bar{g}_{i,t}\bar{g}_{c,t} + \sigma^2 m_{b,t-1}}{\sigma^2_{b,t-1}\sum_{i=1}^{n}\bar{g}^2_{c,t} + \sigma^2} b}{2\frac{\sigma^2_{b,t-1}\sigma^2}{\sigma^2_{b,t-1}\sum_{i=1}^{n}\bar{g}^2_{c,t} + \sigma^2}}\right).\end{aligned}$$

In the exponential function, the denominator is

$$\begin{aligned}\frac{\sigma^2_{b,t-1}\sigma^2}{\sigma^2_{b,t-1}\sum_{i=1}^{n}\bar{g}^2_{c,t} + \sigma^2} &= \frac{1}{\sum_{i=1}^{n}\bar{g}^2_{c,t}/\sigma^2 + 1/\sigma^2_{b,t-1}}\\ &= \frac{1}{\underbrace{n\sigma_c^2\eta_t^2/\sigma^2}_{=\kappa_t} + 1/\sigma^2_{b,t-1}} = \frac{1}{1/\sigma^2_{b,t}} = \sigma^2_{b,t}.\end{aligned}$$

The second term in the numerator is

$$\begin{aligned}&\frac{\sigma^2_{b,t-1}\sum_{i=1}^{n}\bar{g}_{i,t}\bar{g}_{c,t} + \sigma^2 m_{b,t-1}}{\sigma^2_{b,t-1}\sum_{i=1}^{n}\bar{g}^2_{c,t} + \sigma^2}\\ &= \frac{\left(\sum_{i=1}^{n}\bar{g}^2_{c,t}\right)^{-1}\sum_{i=1}^{n}\bar{g}_{i,t}\bar{g}_{c,t} \times \kappa_t + m_{b,t-1}/\sigma^2_{b,t-1}}{\kappa_t + 1/\sigma^2_{b,t-1}}\\ &= \kappa_t\sigma^2_{b,t}\hat{b}_t + \left(1 - \kappa_t\sigma^2_{b,t}\right)m_{b,t-1} = m_{b,t},\end{aligned}$$

where we have defined $\hat{b}_t = (\sum_{i=1}^n \bar{g}_{c,t}^2)^{-1} \sum_{i=1}^n \bar{g}_{i,t}\bar{g}_{c,t}$. We can now express the Bayesian posterior as

$$\text{Prob}\left(b|\bar{g}_1, \bar{g}_2, ..., \bar{g}_t, \bar{g}_{c,1}, \bar{g}_{c,2}, ..., \bar{g}_{c,t}\right) \propto \exp\left(-\frac{(b-m_{b,t})^2}{2\sigma_{b,t}^2}\right).$$

Appendix B. Numerical solution

To solve the firm's problem (13) numerically, we first state the value function in terms of stationary variables. To this end, we define

$$k_{i,t} = \frac{K_{i,t}}{X_{i,t}}, \qquad \tau_{i,t} = \frac{K_{i,t+1}}{X_{i,t}}, \qquad i_{i,t} = \frac{I_{i,t}}{X_{i,t}}.$$

As a result, the stationary version of the law of motion for capital becomes $\tau_{i,t} = (1-\delta)k_{i,t} + i_{i,t}$ and the stationary value function is given by

$$v(s_{i,t}) = \max_{\tau_{i,t}} \left\{ k_{i,t}^\alpha - i_{i,t} - \frac{\phi}{2}\left(\frac{i_{i,t}}{k_{i,t}}\right)^2 k_{i,t} + (1-\pi)\tilde{\mathbb{E}}_t\left[M_{t+1}e^{g_{i,t+1}}v(s_{i,t+1})\right] \right\},$$

where $s_{i,t} = (k_{i,t}, m_{b,t}, \sigma_{b,t})$ is the vector of state variables for the stationary problem. Since the agent does not know the true b, she forms Bayesian beliefs over b denoted by $b_{t+1|t} \sim N(m_{b,t}, p_{b,t})$. Consequently, the perceived productivity process follows

$$\tilde{g}_{i,t+1} = \mu + b_{t+1|t}\sigma_c\eta_{t+1} + \sigma\varepsilon_{i,t+1}.$$

We solve this firm problem using the value function iteration, in which we discretize the grid for capital with 100 points, the average Bayesian belief with 15 points, and the dispersion with 5 points. To compute the continuation value of the value function, we use a Gauss-Hermite quadrature with 5 points to integrate over a 4-dimensional space: the idiosyncratic $\varepsilon_{i,t+1}$ and aggregate shock η_{t+1}, the subjective distribution over risk exposure $b_{t+1|t}$, and the shock distribution of peer firms. Specifically, the mean belief can be written as

$$m_{b,t} = \sigma_{b,t}^2\left(\frac{m_{b,t-1}}{\sigma_{b,t-1}^2} + \frac{g_{c,t}\left[bg_{c,t} + \sigma\varepsilon_{i,t} + (n-1)bg_{c,t} + \sigma\sum_{j\neq i}\varepsilon_{j,t}\right]}{\sigma^2}\right),$$

where n is the number of constituents in the industry. This result implies that firm i's updating of $m_{b,t}$ depends on its peers' idiosyncratic shocks $\varepsilon_{j,t}$. However, only the sum of its peers' idiosyncratic shocks matters. This sum is normally distributed $\sum_{j\neq i}\varepsilon_{j,t} \sim N\left(0, \sqrt{n-1}\right)$, because peers' idiosyncratic shocks are independent of each other and follow a standard normal distribution.

Appendix C. Analytic solutions

Proof of Lemma 1. When the risk exposure parameter is known and capital adjustment is frictionless, the firm's problem simplifies to

$$v(k_{i,t}) = \max_{\tau_{i,t}} \left\{ k_{i,t}^\alpha - i_{i,t} + (1-\pi)\mathbb{E}_t\left[M_{t+1}e^{g_{i,t+1}}v(\tau_{i,t}e^{-g_{i,t+1}})\right] \right\}.$$

The envelope condition is

$$v'(k_{i,t}) = \alpha k_{i,t}^{\alpha-1} + (1-\delta).$$

Integrating $v(k_{i,t})$ with respect to $k_{i,t}$, we solve for the firm value

$$v(k_{i,t}) = \int v'(k_{i,t})dk_{i,t} = k_{i,t}^\alpha + (1-\delta)k_{i,t} + c,$$

where c is a constant to be determined.

The first-order condition is

$$0 = -1 + (1-\pi)\mathbb{E}_t\left[M_{t+1}e^{g_{i,t+1}}v'\left(\tau_{i,t}e^{-g_{i,t+1}}\right)e^{-g_{i,t+1}}\right].$$

Thus, optimal capital $\tau_{i,t}^*$ is given by

$$\tau^* = \underbrace{\left[\frac{\alpha(1-\pi)}{1-(1-\pi)(1-\delta)e^{-r_f}}\right]^{1/(1-\alpha)} e^{-r_f/(1-\alpha)-0.5\gamma^2/(1-\alpha)+\mu+0.5(1-\alpha)\sigma^2}}_{\equiv\chi} \times e^{0.5\left[(1-\alpha)b\sigma_c-\gamma\right]^2/(1-\alpha)}.$$

Note that $\chi > 0$ for $0 < \alpha < 1$ and $0 < \delta < 1$. To see the dependence of τ^* on b, we differentiate τ^* with respect to b

$$\frac{\partial\tau^*}{\partial b} = \chi e^{0.5\left[(1-\alpha)b\sigma_c-\gamma\right]^2/(1-\alpha)}\sigma_c\left[(1-\alpha)b\sigma_c-\gamma\right].$$

This derivative is negative if $b < \gamma/\left[(1-\alpha)\sigma_c\right]$.

Next, we can solve for the unknown constant c by plugging the functional form of $v(\tau^* e^{-g_{i,t+1}})$ into the firm's problem. As a result, we obtain

$$c = \frac{\overbrace{\left[-1+1/\alpha+e^{-r_f}(1-\pi)(1-\delta)(1-1/\alpha)\right]}^{\equiv\kappa_1}\tau^*}{1-(1-\pi)\mathbb{E}_t\left[M_{t+1}e^{g_{i,t+1}}\right]}.$$

Tobin's Q is the ex-dividend firm value divided by capital

$$q_{i,t} = \frac{v(k_{i,t}) - k_{i,t}^\alpha - (1-\delta)k_{i,t} + \tau^*}{\tau^*} = \frac{\kappa_1}{1-\underbrace{(1-\pi)e^{-r_f-0.5\gamma^2+\mu+0.5\sigma^2}}_{\equiv\kappa_2}e^{0.5(b\sigma_c-\gamma)^2}} + 1.$$

This result implies that the Tobin's Q is constant, which we denote by q. Also, note that κ_1 and κ_2 are all positive for $0 < \alpha < 1$ and $0 < \delta < 1$. The derivative of $q_{i,t}$ with respect to b is

$$\frac{\partial q}{\partial b} = \frac{\kappa_1\kappa_2 e^{0.5(b\sigma_c-\gamma)^2}(b\sigma_c-\gamma)\sigma_c}{\left[1-\kappa_2 e^{0.5(b\sigma_c-\gamma)^2}\right]^2}.$$

This derivative is negative if $b < \gamma/\sigma_c$. In addition, q is positive only if $e^{(b\sigma_c-\gamma)^2/2} < 1/\kappa_2$ or $e^{(b\sigma_c-\gamma)^2/2} > (1+\kappa_1)/\kappa_2$. Thus, in the following analysis, we only consider the range of b values that satisfy this condition.

Proof of Lemma 2. The expected gross return is

$$\mathbb{E}_t\left[R_{i,t}\right] = \mathbb{E}_t\left[\frac{(1-\pi)V(X_{i,t+1}, K_{i,t+1})}{V(X_{i,t}, K_{i,t}) - D_{i,t}}\right] = (1-\pi)\frac{\mathbb{E}_t\left[(\tau^*)^{\alpha-1}e^{(1-\alpha)g_{i,t+1}} + (1-\delta) + \frac{c}{\tau^*}e^{g_{i,t+1}}\right]}{q}.$$

Let $N(b)$ denote the numerator of the expected return expression, which we expand as follows

$$N(b) = \mathbb{E}_t\left[\left(\tau^*\right)^{\alpha-1}e^{(1-\alpha)g_{i,t+1}} + (1-\delta) + \frac{c}{\tau^*}e^{g_{i,t+1}}\right] = \underbrace{\frac{1-(1-\delta)e^{-r_f}}{\alpha}}_{\equiv\kappa_3}e^{r_f+(1-\alpha)b\sigma_c\gamma} + (1-\delta)\frac{\kappa_1 e^{\mu+0.5b\sigma_c^2+0.5\sigma^2}}{1-\kappa_2 e^{0.5(b\sigma_c-\gamma)^2}}.$$

Combining the expressions for $N(b)$ and q, we obtain the expected return as

$$\mathbb{E}_t\left[R_{i,t+1}\right] = (1-\pi)\frac{N(b)}{q}$$

$$= (1-\pi)\frac{1-\kappa_2 e^{0.5(b\sigma_c-\gamma)^2}}{\kappa_1+1-\kappa_2 e^{0.5(b\sigma_c-\gamma)^2}} \times \left[\kappa_3 e^{r_f+(1-\alpha)b\sigma_c\gamma} + (1-\delta) + \frac{\kappa_1 e^{\mu+0.5b\sigma_c^2+0.5\sigma^2}}{1-\kappa_2 e^{0.5(b\sigma_c-\gamma)^2}}\right].$$

The derivative of the expected return with respect to b is

$$\frac{\partial \mathbb{E}_t\left[R_{i,t+1}\right]}{\partial b} = (1-\pi)\frac{\kappa_3 e^{r_f+(1-\alpha)b\sigma_c\gamma}(1-\alpha)\gamma\sigma_c + b\sigma_c^2 \frac{\kappa_1 e^{\mu+b^2\sigma_c^2/2+\sigma^2/2}}{1-\kappa_2 e^{(b\sigma_c-\gamma)^2/2}}}{1+\kappa_1-\kappa_2 e^{(b\sigma_c-\gamma)^2/2}}$$

$$+ (1-\pi)\frac{(1-\delta)(\gamma-b\sigma_c) + \kappa_3 e^{r_f+(1-\alpha)b\sigma_c\gamma}(-b\sigma_c^2+\alpha\gamma\sigma_c) + \frac{\kappa_1 e^{\mu+b^2\sigma_c^2/2+\sigma^2/2}}{1-\kappa_2 e^{(b\sigma_c-\gamma)^2/2}}\left(\sigma_c\gamma - 2b\sigma_c^2\right)}{\left(1+\kappa_1-\kappa_2 e^{(b\sigma_c-\gamma)^2/2}\right)^2} \times \kappa_1\kappa_2 e^{(b\sigma_c-\gamma)^2/2}$$

$$+ (1-\pi)\frac{e^{(b\sigma_c-\gamma)^2/2}\kappa_2\left(1-\kappa_2 e^{(b\sigma_c-\gamma)^2/2}\right)}{\left(1+\kappa_1-\kappa_2 e^{(b\sigma_c-\gamma)^2/2}\right)^2} \times \left[(1-\delta)(1-\sigma_c)(\gamma-b\sigma_c) - b\sigma_c^2\frac{\kappa_1 e^{\mu+b^2\sigma_c^2/2+\sigma^2/2}}{1-\kappa_2 e^{(b\sigma_c-\gamma)^2/2}}\right].$$

When $e^{(b\sigma_c-\gamma)^2/2} < 1/\kappa_2$, it follows that $e^{(b\sigma_c-\gamma)^2/2} < (1+\kappa_1)/\kappa_2$ because $\kappa_1 > 0$. Thus, the derivative of return is positive if b satisfies

$$b \le \max\left(\frac{\alpha\gamma}{\sigma_c}, \frac{\gamma}{2\sigma_c}\right) \quad \text{and}$$

$$(1-\delta)(1-\sigma_c)\gamma \ge (1-\delta)(1-\sigma_c)\sigma_c b + b\sigma_c^2\frac{\kappa_1 e^{\mu+b^2\sigma_c^2/2+\sigma^2/2}}{1-\kappa_2 e^{(b\sigma_c-\gamma)^2/2}}.$$

Appendix D. Learning about dynamic risk exposure

In this section, we formulate the updating of beliefs about risk exposure when the true parameter is stochastic, following an autoregressive process as in equation (17). To facilitate the formulation, we express the law of motion of risk exposure using a state-space representation

$$\underbrace{\begin{bmatrix} b_{t+1} \\ \bar{b} \end{bmatrix}}_{=B_{t+1}} = \underbrace{\begin{bmatrix} \varphi & 1-\varphi \\ 0 & 1 \end{bmatrix}}_{=\Phi} \underbrace{\begin{bmatrix} b_t \\ \bar{b} \end{bmatrix}}_{=B_t} + \sigma_b \begin{bmatrix} \xi_{t+1} \\ 0 \end{bmatrix}. \tag{D.1}$$

Our goal is to update beliefs about B_{t+1} using new observations at time $t+1$ along with the transition equation (D.1). The new observations are the demeaned growth of productivity of industry constituents, which is given by

$$\underbrace{\begin{bmatrix} \bar{g}_{1,t+1} \\ \bar{g}_{2,t+1} \\ \vdots \\ \bar{g}_{n,t+1} \end{bmatrix}}_{=Y_{t+1}} = \underbrace{\begin{bmatrix} \bar{g}_{c,t+1} & 0 \\ \bar{g}_{c,t+1} & 0 \\ \vdots & \vdots \\ \bar{g}_{c,t+1} & 0 \end{bmatrix}}_{=A_{t+1}} \underbrace{\begin{bmatrix} b_{t+1} \\ \bar{b} \end{bmatrix}}_{=B_{t+1}} + \sigma \underbrace{\begin{bmatrix} \epsilon_{1,t+1} \\ \epsilon_{2,t+1} \\ \vdots \\ \epsilon_{n,t+1} \end{bmatrix}}_{=V_{t+1}}. \tag{D.2}$$

We now have the state-space model consisting of the transition equation (D.1) and observation equation (D.2). We next apply the Kalman filter to derive the distribution of B_{t+1} conditional on all observations until time $t+1$. Suppose that B_t conditional on all observations until time t is normally distributed with mean $m_{B,t}$ and covariance $\Sigma_{B,t}$. Then, the covariance between B_{t+1} and Y_{t+1} and the variance of Y_{t+1} are given by[25]

[25] We assume that the consumption growth in A_{t+1} is known prior to updating beliefs at time $t+1$.

$$\mathrm{Cov}_t\left(B_{t+1}, Y_{t+1}\right) = \underbrace{\left(\Phi\Sigma_{B,t}\Phi^\top + \begin{bmatrix} \sigma_b^2 & 0 \\ 0 & 0 \end{bmatrix}\right)}_{=\Sigma_{B,t+1|t}} A_{t+1}^\top$$

$$\mathrm{Var}_t\left(Y_{t+1}\right) = A_{t+1}\Sigma_{B,t+1|t}A_{t+1}^\top + \sigma^2 I_n,$$

where $\Sigma_{B,t+1|t}$ is the prior of the covariance, and I_n is the $(n\times n)$ identity matrix. Because B_{t+1} and Y_{t+1} follow a joint normal distribution, the conditional mean and covariance of B_{t+1} can be updated

$$m_{B,t+1} = \Phi m_{B,t} + \mathrm{Cov}_t\left(B_{t+1}, Y_{t+1}\right)\left[\mathrm{Var}_t\left(Y_{t+1}\right)\right]^{-1}\left(Y_{t+1} - A_{t+1}\Phi m_{B,t}\right)$$
$$= \Phi m_{B,t} + \underbrace{\left[\Sigma_{B,t+1|t}A_{t+1}^\top\right]\left[A_{t+1}\Sigma_{B,t+1|t}A_{t+1}^\top + \sigma^2 I_n\right]^{-1}}_{=\mathbb{K}_{t+1}} \times \left(Y_{t+1} - A_{t+1}\Phi m_{B,t}\right),$$

$$\Sigma_{B,t+1} = \left(I - \mathbb{K}_{t+1}A_{t+1}\right)\Sigma_{B,t+1|t},$$

where $\mathbb{K}_{t+1}$ is the Kalman gain.

This filtering depends on the model parameters. We estimate the parameters (φ, σ_b) for each industry using the expectation-maximization algorithm. Plugging these parameter estimates into the updating equation, we obtain the posterior beliefs about the risk exposure vector B_t. The time-t beliefs are normally distributed with the following conditional mean and covariance:

$$m_{B,t}^{\mathrm{KF}} = \begin{bmatrix} m_{b_t,t}^{\mathrm{KF}} \\ m_{\bar{b},t}^{\mathrm{KF}} \end{bmatrix}, \quad \Sigma_{B,t}^{\mathrm{KF}} = \begin{bmatrix} \left(\sigma_{b_t,t}^{\mathrm{KF}}\right)^2 & \mathrm{Cov}_t^{\mathrm{KF}}\left(b_t, \bar{b}\right) \\ \mathrm{Cov}_t^{\mathrm{KF}}\left(b_t, \bar{b}\right) & \left(\sigma_{\bar{b},t}^{\mathrm{KF}}\right)^2 \end{bmatrix}.$$

Appendix E. Joint learning about productivity drift and risk exposure

In this section, we assume that both drift μ and risk exposure b of productivity are unknown constants and must be learned simultaneously. We formulate this joint learning using a state-space model and estimate parameter beliefs through the Kalman Filter as in section Appendix D. First, we let X denote a column vector of the unknown parameters

$$X = \begin{bmatrix} \mu \\ b \end{bmatrix}.$$

We assume that beliefs about X conditional on all observations until time t are normally distributed with mean $m_{X,t}$ and covariance $\Sigma_{X,t}$. We aim to update the beliefs about X using new observations of productivity at time $t+1$. The cross-sectional observations of productivity are stacked in vector Y_{t+1}, which is given by

$$\underbrace{\begin{bmatrix} g_{1,t+1} \\ g_{2,t+1} \\ \vdots \\ g_{n,t+1} \end{bmatrix}}_{=Y_{t+1}} = \underbrace{\begin{bmatrix} 1 & \bar{g}_{c,t+1} \\ 1 & \bar{g}_{c,t+1} \\ \vdots & \\ 1 & \bar{g}_{c,t+1} \end{bmatrix}}_{=A_{t+1}} \underbrace{\begin{bmatrix} \mu \\ b \end{bmatrix}}_{=X} + \sigma \underbrace{\begin{bmatrix} \epsilon_{1,t} \\ \epsilon_{2,t} \\ \vdots \\ \epsilon_{n,t} \end{bmatrix}}_{=V_{t+1}}. \tag{E.1}$$

We now have a state-space model with observation equation (E.1) but without transition in the underlying state X. Next, as in section Appendix D, we apply the Kalman filter to update the distribution of X conditional on new observations at time $t+1$. The key inputs to the Kalman filter are

$$\mathrm{Cov}_t\left(X, Y_{t+1}\right) = \Sigma_{X,t}A_{t+1}^\top$$
$$\mathrm{Var}_t\left(Y_{t+1}\right) = A_{t+1}\Sigma_{X,t}A_{t+1}^\top + \sigma^2 I_n.$$

Finally, the conditional mean and covariance of X are updated as following

$$m_{X,t+1} = m_{X,t} + \mathrm{Cov}_t\left(X, Y_{t+1}\right)\left[\mathrm{Var}_t\left(Y_{t+1}\right)\right]^{-1}\left(Y_{t+1} - A_{t+1}m_{X,t}\right)$$

$$= m_{X,t} + \underbrace{\left[\Sigma_{X,t} A_{t+1}^{\top}\right]\left[A_{t+1}\Sigma_{X,t}A_{t+1}^{\top} + S\right]^{-1}}_{=\mathbb{K}_{t+1}} \left(Y_{t+1} - A_{t+1} m_{X,t}\right)$$

$$\Sigma_{X,t+1} = \left(I - \mathbb{K}_{t+1} A_{t+1}\right)\Sigma_{X,t}.$$

Specifically, the time-t beliefs are normally distributed with the following conditional mean and covariance

$$m_{X,t} = \begin{bmatrix} m_{\mu,t}^{\mathrm{KF}} \\ m_{b,t}^{\mathrm{KF}} \end{bmatrix}, \quad \Sigma_{X,t} = \begin{bmatrix} \left(\sigma_{\mu,t}^{\mathrm{KF}}\right)^2 & \mathrm{Cov}_t^{\mathrm{KF}}(\mu, b) \\ \mathrm{Cov}_t^{\mathrm{KF}}(\mu, b) & \left(\sigma_{b,t}^{\mathrm{KF}}\right)^2 \end{bmatrix}.$$

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