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Too Levered for Pigou: Carbon pricing, financial constraints, and leverage regulation☆

Robin Döttling [a,c], Magdalena Rola-Janicka [b,c,d],*

[a] *Erasmus University Rotterdam, Burgemeester Oudlaan 50, 3062 PA, Rotterdam, The Netherlands*
[b] *Imperial College London, 53 Prince's Gate, SW7 2PH, London, United Kingdom*
[c] *Finance Theory Group, United States of America*
[d] *CEPR, 187 Boulevard Saint Germain, 75007 Paris, France*

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ABSTRACT

We analyze optimal carbon pricing under financial constraints and endogenous climate-related transition and physical costs. The socially optimal emissions tax may be above or below a Pigouvian benchmark, depending on the strength of physical climate impacts on pledgeable resources. We derive necessary conditions for emissions taxes alone to implement a constrained-efficient allocation, and show a cap-and-trade system may dominate emissions taxes because it can be designed to have a less adverse effect on financial constraints. We also assess how capital structure, carbon price hedging markets, and socially responsible investors interact with emissions pricing, and evaluate other commonly used policy tools.

1. Introduction

Tackling climate change requires large-scale emissions reductions and investments in clean technologies. Absent frictions, such investments can be incentivized through emissions taxes set at a rate equal to the marginal social cost of emissions, also known as Pigouvian taxes in reference to Pigou (1920). However, during the transition to a low-carbon economy firms and financial institutions may suffer significant losses that can exacerbate the severity of financial frictions. Such frictions can limit the ability of firms to make the necessary investments in green technologies and constrain regulators in designing environmental policies. Consistent with this notion, recent empirical evidence underlines the relevance of financial frictions for firms' green

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investments (Xu and Kim, 2022; Kacperczyk and Peydró, 2022; Martinsson et al., 2024a), and the risks posed by climate change have moved up the agenda of investors and financial policy makers (Brunnermeier and Landau, 2022; Krueger et al., 2020). Motivated by these considerations, this paper evaluates how carbon pricing policies interact with financial constraints.

A natural intuition is that the presence of financial constraints implies lower optimal emissions taxes because high taxes, costly abatement investments, and other climate transition-related costs tighten financial constraints. Our first key result shows that these effects may reverse once we account for the interaction between financial constraints and the negative impact of a warming climate on borrowers' assets. This interaction gives rise to a climate-induced collateral externality through which emissions taxes can relax, rather than tighten, financial constraints. This collateral externality motivates higher optimal emissions taxes, and if it is sufficiently strong, it may even imply that financial constraints lead to an optimal tax rate above a standard Pigouvian benchmark.

Our second key contribution lies in evaluating the relative merit of carbon taxes and cap-and-trade systems in the presence of financial constraints. We show that a cap-and-trade system may dominate a carbon tax, but that this crucially depends on how the regulator allocates pollution permits, highlighting the importance of understanding the nuances of how financial constraints interact with environmental policy design. We also identify conditions under which combining carbon pricing with other policy tools can improve welfare, and demonstrate how carbon pricing could be complemented by leverage regulation. Additionally, we show that efficient carbon pricing can be supported by carbon price hedging markets. Moreover, socially responsible investors can either support or hinder carbon pricing policies, depending on the nature of investors' social preferences.

We derive these insights in a model with two dates and two types of agents: borrowers and deep-pocketed lenders. Each borrower has an initial investment project in place, which generates a pecuniary return as well as carbon emissions at the final date. A regulator sets an emissions tax in order to incentivize borrowers to reduce their emissions through costly abatement activities. Borrowers need to raise debt to finance abatement spending, but debt issuance is limited by a financial constraint because project returns are not fully pledgeable to outside investors. To capture the notion of "stranded assets", we assume that constrained borrowers can liquidate part of the initial investment to generate resources and reduce emissions, yet liquidations are inefficient due to liquidation losses.

In the model, all agents suffer disutility due to *aggregate* emissions, but agents do not internalize the impact of their actions on aggregate variables because they are atomistic. A key feature of our model is that we also assume that the payoffs of borrowers' projects may decrease in the level of aggregate emissions. In the Internet Appendix, we provide different micro foundations for this assumption. Our preferred interpretation is that it captures the negative effect of physical climate risk on asset values or direct negative cash flow effects due to extreme weather events. Such effects have been extensively documented in the empirical literature (Ginglinger and Moreau, 2023; Issler et al., 2020; Giglio et al., 2021b; Pankratz and Schiller, 2024), yet the interaction of these effects with financial constraints and implications for optimal emissions pricing have not been explored in the theoretical literature.[1]

Thus, in our model borrowers' projects are exposed to the two major categories of climate-related costs laid out by the Task Force on Climate-Related Financial Disclosures (TCFD, 2017): (1) emissions taxes and abatement costs resemble costs associated with the transition to a low-carbon economy, and (2) losses to borrowers' assets due to aggregate emissions resemble costs related to the physical impacts of climate change. Both climate-related costs are endogenous in the model: transition-related costs are a consequence of emissions taxes optimally set by an environmental regulator, and losses due to physical climate impacts are a function of abatement and investment decisions by (other) borrowers. This allows us to explore the differences in how these two types of climate-related costs interact with financial frictions and affect optimal environmental and financial policies in equilibrium.

As a benchmark, we show that an emissions tax equal to the social cost of emissions (i.e., a Pigouvian benchmark tax) implements the first-best allocation if the financial constraint is slack. By contrast, if financial constraints bind, optimal emissions taxes generally differ from the Pigouvian benchmark. The reason is that constrained borrowers have a limited ability to finance abatement and therefore need to inefficiently liquidate some of the initial investment. Consequently, the socially optimal emissions tax needs to trade off the benefit of lower emissions against the cost of triggering inefficient liquidations. In the absence of physical climate impacts on borrowers' pledgeable assets, a higher emissions tax bill and greater abatement spending unambiguously tighten financial constraints, motivating an optimal emissions tax below the Pigouvian benchmark in line with common intuition.

A key insight from our analysis is that physical climate impacts can reverse the relationship between emissions taxes and financial constraints. This is because a higher emissions tax reduces aggregate emissions, which can lead to an increase in pledgeable resources for borrowers whose assets are exposed to physical climate impacts. Given that this mechanism operates through the effect of aggregate emissions on pledgeable resources, we refer to it as a *climate-induced collateral externality*. This collateral externality arises endogenously due to the interaction of physical climate impacts and financial constraints, resulting in higher optimal emissions taxes. If the collateral externality is sufficiently strong, it may even motivate an optimal emissions tax *above* the Pigouvian benchmark rate. More broadly, we show that financial constraints call for a generalized Pigouvian tax that takes climate-induced collateral externalities into account. This generalized Pigouvian benchmark rate increases in the severity of physical climate impacts, especially if financial constraints are very tight. This underscores the importance of understanding how financial constraints interact with environmental policy: a regulator overlooking this interaction would set sub-optimally low emissions taxes.

Given that financial constraints distort emissions pricing and give rise to a collateral externality, a natural question is whether a regulator can do better by using additional policy tools. Our second key result answers this question: despite the presence of financial constraints, emissions taxes alone can implement a constrained-efficient allocation without relying on other policy tools — but only if the tax proceeds from emissions taxes are fully rebated to borrowers and these tax rebates are fully pledgeable to outside investors.[2]

However, the pledgeability of tax rebates to outside financiers may be beyond the control of the regulator, for instance due to limitations of political or legal institutions. We show that if tax rebates are not fully pledgeable, replacing carbon taxes with a cap-and-trade system with tradable pollution permits, such as the EU Emissions Trading System, may enhance welfare. (We further examine emissions caps, non-linear taxes, and green subsidies in the Internet Appendix.) Previous literature demonstrates that, in the absence of frictions, cap-and-trade systems and emissions taxes yield equivalent outcomes, with the initial

[1] Giglio et al. (2021b) find that the value of real estate in flood zones responds more to changes in climate attention. Issler et al. (2020) document an increase in delinquencies and foreclosures after wildfires in California. Evidence in Ginglinger and Moreau (2023) indicates that physical climate risks affect a firm's capital structure. For a review discussing climate risks, see Giglio et al. (2021a).

[2] Examples of such tax rebates in practice include the "Canada Carbon Rebate" and the Austrian "Klimabonus". Constrained efficiency requires that such rebates are fully pledgeable to outside investors.

allocation of permits being irrelevant (see Montgomery, 1972). This is known as the *independence property*. Independence breaks down under frictions such as transaction costs or market power, typically leading to increased emissions when more permits are allocated for free (Hahn, 1984; Stavins, 1995; Fowlie and Perloff, 2013; Fowlie et al., 2016). We show that under financial constraints the failure of the independence property can imply that allocating more free permits results in *lower* emissions. This occurs because allocating more free permits limits the direct negative impact of the cap-and-trade system on pledgeable income. Importantly, we show that a regulator can eliminate the direct impact on pledgeable income altogether by allocating all permits for free. This policy design enables implementing a constrained-efficient carbon price, effectively mirroring emissions taxes with fully pledgeable tax rebates.

These results imply that a cap-and-trade system may be a superior policy tool compared to a carbon tax, but that its efficacy hinges on the regulator allocating permits in a manner that minimizes adverse effects on financial constraints. This is an important consideration given that real-world cap-and-trade systems typically do not allocate all permits for free. For example, the EU is gradually reducing the amount of free permits over time. Our result caveats that under financial constraints this approach can have negative side effects.

The result also speaks to debates on whether, for example, financial policy makers should use policy tools to target climate-related goals. In our model, there is a pecking order for optimal policy tools under financial constraints: only if carbon pricing policies cannot be designed to have a minimal impact on pledgeable income, there is a case to use other policy tools to target climate-related goals.

To directly study financial policy in the model, we introduce an ex-ante capital structure decision. This allows us to examine the interaction between carbon pricing and leverage regulation. A borrower's leverage affects emissions by influencing financial constraints, liquidations, and abatement activities. When tax rebates are fully pledgeable, optimal emissions taxes ensure that emissions externalities are correctly internalized, aligning borrowers' leverage choices with socially efficient levels. Consequently, in this case carbon pricing alone is sufficient to achieve constrained efficiency, as in the baseline model. However, when tax rebates are not fully pledgeable, a gap persists between the social and private costs of emissions, leading to inefficient leverage choices. In such cases, leverage regulation can improve welfare. Interestingly, we find that the socially-optimal leverage may be either above or below the level chosen privately by borrowers, depending on whether borrowers respond to financial constraints mostly by scaling down abatement or by liquidating assets.

In additional analyses, we study how financial markets interact with emissions pricing. In a model extension with climate risk, we show that carbon price hedging markets can enable the regulator to set a more efficient environmental policy in equilibrium, thereby increasing welfare beyond the first-order risk-sharing benefits for borrowers. This highlights an important role the financial sector can play in the transition to a low-carbon economy, distinct from socially responsible investing that aims to reduce emissions by taking environmental and social factors into account in investment decisions. In another extension, we consider socially responsible investors directly in the model. Such investors can provide incentives to reduce emissions by making financing costs contingent on emissions reductions. We show that such an investment strategy can be beneficial if it results in lower financing costs for constrained borrowers, but can have a perverse effect if it increases financing costs and tightens borrowers' financial constraints, consistent with evidence in Kacperczyk and Peydró (2022). This implies that socially responsible investors may be an imperfect substitute for a well-designed carbon pricing policy. Finally, we consider a setting with multiple jurisdictions in which regulators do not internalize the effects of their policy on the rest of the world. Our model highlights that the climate-induced collateral externality gives rise to financial spillovers between jurisdictions, amplifying the costs of inadequate global coordination on climate policy.

This paper relates to several recent contributions that study environmental externalities and green investment under financial and other economic frictions (Tirole, 2010; Biais and Landier, 2022; Huang and Kopytov, 2023; Allen et al., 2023; Lanteri and Rampini, 2023; Inderst and Opp, 2025; Gupta and Starmans, 2024). Hoffmann et al. (2017) and Oehmke and Opp (2025) also show that, in the presence of binding financial constraints, the optimal emissions tax may be below a Pigouvian benchmark. Our analysis contributes by uncovering a novel climate-induced collateral externality that alters the interaction between environmental policy and financial constraints, potentially motivating emissions taxes above a Pigouvian benchmark. The collateral externality-based mechanism also distinguishes our findings from Simpson (1995) and Heider and Inderst (2022), who show that high emissions taxes can be beneficial in models with product market competition as they can shift production to more efficient firms. Our mechanism is related to collateral externalities that arise in models with pecuniary externalities (Stein, 2012; Dávila and Korinek, 2018; Jeanne and Korinek, 2020), but it is distinct in that it operates through losses caused by aggregate emissions rather than mispricing due to fire sales. It also provides distinct implications: fire sale losses amplify financial constraints, which in the absence of physical climate impacts on collateral values motivates lower emissions taxes. By contrast, the climate-induced collateral externality we identify motivates higher emissions taxes.

Our analysis also contributes by evaluating the relative merit of emissions taxes and cap-and-trade systems under financial constraints, and by deriving a necessary condition under which these policy tools can implement a constrained-efficient allocation. By studying financial policy as an additional tool in this context we relate to recent contributions by Oehmke and Opp (2023) and Dávila and Walther (2022), who consider risk-weighted capital regulation as a tool for tackling environmental externalities, as well as to Zhang et al. (2025), who explore income taxes on capital investment as a tool that can complement a carbon tax. We follow a different approach in that we take optimal emissions taxes as a starting point and show under what conditions financial policy, in the form of leverage regulation, can be valuable as a complementary policy tool.

Finally, our model also relates to the literature on socially responsible investing (Heinkel et al., 2001; Chowdhry et al., 2019; Pástor et al., 2021; Green and Roth, 2025; Broccardo et al., 2022; Gupta et al., 2025; Goldstein et al., 2022; Piccolo et al., 2022; Edmans et al., 2022; Hong et al., 2023; Oehmke and Opp, 2025; Geelen et al., 2024). Our findings highlight the role of carbon price hedging markets as a distinct mechanism through which financial markets can facilitate emissions reductions, driven by the positive impact of hedging on equilibrium carbon pricing. We also contribute by investigating the interaction between socially responsible investing and carbon pricing and highlight how socially responsible investors may hinder efficient emissions pricing in equilibrium.

2. Model setup

There are two dates, $t = 1, 2$, a unit mass of investors, and a unit mass of borrowers. Investors are risk-neutral and deep-pocketed in that they have a large endowment A^i at $t = 1$, while borrowers have no endowment and need to raise financing from investors. There is no discounting and all agents suffer disutility from aggregate carbon emissions E^a at $t = 2$. That is, agents' utility is given by

$$U = c_1 + c_2 - \gamma^u E^a, \tag{1}$$

where c_t denotes consumption and γ^u is a parameter governing the cost of emissions in agents' utility. Agents are atomistic, so that they do not internalize the effect of their decisions on aggregate emissions E^a. In the baseline model, we abstract from heterogeneity among borrowers, but we show in the Internet Appendix (Section IA.3) that our main insights hold also in the case of heterogeneous borrowers.

Technology. Borrowers start with an initial investment project in place, representing polluting legacy assets. The initial investment scale is I_0. At $t = 1$, borrowers can liquidate some of the initial assets and adjust the scale to $I_1 \leq I_0$. The project generates a payoff of $R(I_1, E^a)$ at $t = 2$, which increases in I_1, and liquidations generate a payoff $\mu(I_0 - I_1)$ at $t = 1$, with $\mu < 1$.

The project emits carbon emissions $E(X, I_1)$ at $t = 2$, which can be reduced by non-verifiable abatement investments X at a cost $C(X, I_1)$ paid at $t = 1$. Emissions also increase in the final investment scale I_1. Thus, borrowers can reduce emissions either through abatement or through liquidations.

Aggregating over borrowers, emissions aggregate to $E^a = \int_0^1 E(X, I_1)$. A key modeling element is that we assume the payoff $R(I_1, E^a)$ decreases in aggregate emissions E^a. To simplify notation, we refer to the marginal effect of aggregate emissions on payoffs as $\gamma^p \equiv -\partial R(I_1, E^a)/\partial E^a$. Modeling the payoff-relevant effect γ^p allows us to distinguish it from the utility-related component γ^u in comparative statics exercises. As we show in our analysis, this distinction is important because γ^p and γ^u interact differently with financial constraints. The total social cost of emissions consists of the two components and is given by $\gamma = 2\gamma^u + \gamma^p$.

In the Internet Appendix (Section IA.1), we provide different micro foundations for γ^p. Our preferred interpretation is that it captures the negative effect of aggregate emissions on payoffs due to (expected) losses from extreme weather. This may be driven by direct exposure to extreme weather events in the near future, or by negative asset pricing effects due to expected losses from extreme weather over longer horizons. Such asset pricing effects have been documented in the literature (for a review, see Giglio et al., 2021a), and are often described as *physical climate risk*. Since there is no risk in our baseline model, we simply refer to γ^p as "physical climate impacts" and interpret it as capturing the direct and indirect (asset pricing) effects of (expected) extreme weather events that become more likely as aggregate emissions rise and the climate heats up. In the Internet Appendix, we model such asset pricing effects explicitly by assuming the firm sells some of its assets in secondary markets at $t = 2$, and show that the price it can fetch declines in E^a as buyers anticipate greater expected losses in the future. We also show that γ^p could similarly capture other financial spillovers from emissions reductions, such as technological spillovers from developing green technologies.

We make the following functional form assumptions.

Assumption 1. $R(I_1, E^a)$, $E(X, I_1)$ and $C(X, I_1)$ satisfy

1. $\partial C(X, I_1)/\partial X \geq 0$, $\partial C(X, I_1)/\partial I_1 \geq 0$, $\partial E(X, I_1)/\partial X \leq 0$, $\partial E(X, I_1)/\partial I_1 \geq 0$, $\partial R(I_1, E^a)/\partial I_1 \geq 0$, $\partial R(I_1, E^a)/\partial E^a \leq 0$,
2. $C(0, I_1) = 0$, $C(X, 0) = 0$, $\lim_{X\to\infty} E(X, I_0) = 0$, $E(X, 0) = 0$, $E(0, I_0) = \bar{E}$,
3. $\partial^2 E(X, I_1)/\partial X^2 = 0$, $\partial^2 C(X, I_1)/\partial X^2 > 0$.

Assumption 1.1 ensures that abatement investments are costly but reduce emissions, and that a higher final investment scale is associated with higher emissions, payoffs, and abatement costs. Assumption 1.2 defines boundaries such that costs and emissions are non-negative, and there is an upper bound $\bar{E}$ on emissions. Assumption 1.3 implies that emissions are linear in abatement, which simplifies the exposition, and that the cost of abatement is strictly convex, so that the borrower's optimal abatement choice has an interior solution.

Environmental regulation. An environmental regulator sets an emissions tax τ per unit of emissions in order to maximize social welfare.[3] Emissions taxes are rebated lump-sum to borrowers, $T = \tau E^a$ (such tax rebates are sometimes referred to as a "carbon dividend" in policy debates). Section 4.3 studies a cap-and-trade system as an alternative policy tool, while quantity limits on emissions and green subsidies are covered in the Internet Appendix.

Financing. At $t = 1$, borrowers can finance abatement X by raising debt d_1 from investors. External financing is limited by a moral hazard problem. We assume that at $t = 2$ borrowers can abscond with any resources except a fraction $\theta \in [0, 1]$ of asset payoffs, and a fraction $\psi \in [0, 1]$ of tax rebates T. Thus, there is a wedge between the project's payoff and pledgeable income, with pledgeable payoffs given by $\theta R(I_1, E^a)$ (as in Rampini and Viswanathan, 2013, among others). The separate pledgeability parameter for tax rebates allows us to perform key comparative statics exercises. For example, when $\psi = 1$ and $\theta < 1$, borrowers may be financially constrained but tax rebates are fully pledgeable.

In the baseline model, borrowers have no outstanding debt at the beginning of $t = 1$. In reality, past financing choices may also affect borrowers' financial constraints and ability to finance abatement. In Section 5.1, we extend the model to study the ex-ante financing choice of the initial investment I_0 by introducing an initial stage $t = 0$. This enables us to study the role of ex-ante leverage choices and regulation used alongside carbon pricing policies. Building on this extension, we also introduce ex-ante uncertainty about the social cost of carbon γ, which allows us to study the role of other types of financing such as carbon-price-hedging contracts, risky long-term debt, and equity. Moreover, we explore the effect of introducing socially responsible investors. These extensions provide interesting additional insights on how financial markets interact with equilibrium environmental policy.

Variable definitions. For the further analysis it will be useful to introduce the following variable definitions and assumptions:

Definition 1. The project's private net marginal return on investment, denoted by $r(\tau, X, I_1)$, and pledgeable net marginal return on investment, denoted by $\tilde{r}(\tau, X, I_1)$, are defined as, respectively,

$$r(\tau, X, I_1) = \frac{\partial R(I_1, E^a)}{\partial I_1} - \mu - \frac{\partial C(X, I_1)}{\partial I_1} - \tau \frac{\partial E(X, I_1)}{\partial I_1},$$

$$\tilde{r}(\tau, X, I_1) = \theta \frac{\partial R(I_1, E^a)}{\partial I_1} - \mu - \frac{\partial C(X, I_1)}{\partial I_1} - \tau \frac{\partial E(X, I_1)}{\partial I_1}.$$

Assumption 2. Project payoffs are sufficiently large and pledgeability θ sufficiently small such that, given a threshold $\bar{\tau} \geq \gamma$,

1. $r(\tau, X, I_1) > 0, \ \forall X, I_1, \tau \leq \bar{\tau}$,
2. $\tilde{r}(0, X, I_1) < 0, \ \forall X, I_1$.

In principle, sufficiently high emissions taxes can always render the project non-viable. The first condition ensures that continuing rather than liquidating the project has a positive NPV as long as emissions taxes do not exceed some threshold $\bar{\tau}$. To ensure continuation of the project is privately profitable, we focus on $\tau \leq \bar{\tau}$ throughout the paper. While in reality a mix of liquidations and abatement may be optimal, we assume that liquidations are inefficient to cleanly distinguish between efficient abatement spending and inefficient liquidations. The second condition ensures that, while inefficient, liquidations relax financial constraints.

[3] In the baseline model we only consider a linear tax because there is no heterogeneity among borrowers. In Internet Appendix IA.2 we discuss non-linear taxes as well as quantity-based regulation (an emissions cap) that fixes the level of abatement, which is equivalent to a non-linear tax that is zero below a threshold level and infinite above the threshold. In Internet Appendix IA.3 we allow for borrower heterogeneity and discuss non-linear taxes in this context.

2.1. First-best benchmark

Proposition 1. *In the first-best allocation there are no liquidations, $I_1 = I_0$, and the optimal abatement X solves*

$$\gamma \frac{\partial E(X, I_1)}{\partial X} = -\frac{\partial C(X, I_1)}{\partial X}. \tag{2}$$

Proof. See Appendix A.1 □

In the first-best allocation, the optimal abatement equates the marginal gain from lower emissions with the marginal cost of abatement. Crucially, there are no liquidations because liquidations are inefficient by Assumption 2. The next section shows that this may be different in the competitive equilibrium, where financially constrained borrowers may need to liquidate some of their initial investment.

3. Competitive equilibrium

This section solves the problem of borrowers and defines a competitive equilibrium given an emissions tax τ. We analyze optimal emissions taxes and other policy tools in later sections.

3.1. Borrower problem

Borrowers maximize their utility subject to the following constraints:

$$c_1 + C(X, I_1) = (I_0 - I_1)\mu + d_1, \tag{3}$$

$$c_2 + d_1 + \tau E(X, I_1) = R(I_1, E^a) + T, \tag{4}$$

$$d_1 \leq \theta R(I_1, E^a) - \tau E(X, I_1) + \psi T, \tag{5}$$

$$c_t \geq 0, \ \ t = 1, 2, \tag{6}$$

$$I_1 \in [0, I_0]. \tag{7}$$

Eqs. (3) and (4) are budget constraints at $t = 1$ and $t = 2$. Eq. (5) is a financial constraint that ensures debt does not exceed pledgeable income. It follows from the incentive-compatibility condition $c_2 \geq (1-\theta)R(I_1, E^a) + (1-\psi)T$ and implies borrowers have no incentive to abscond at $t = 2$. Finally, (6) denotes the non-negativity constraints on consumption at each date, and (7) puts bounds on the investment scale.

Using the budget constraints to eliminate c_1, c_2 and d_1, the borrower's problem can be formulated as a Lagrange function of X and I_1, with Lagrange multipliers λ for the $t = 1$ financial constraint, and $\overline{\kappa}_I$, $\underline{\kappa}_I$ serving as multipliers for upper and lower bounds on investment scale. The Lagrangian is formally stated in Eq. (A.2) in Appendix A.2.

3.2. Borrower decisions

Borrowers observe the tax τ and then choose abatement X and liquidations $I_0 - I_1$ according to the following conditions.

$$(1+\lambda)\left(\tau \frac{\partial E(X, I_1)}{\partial X} + \frac{\partial C(X, I_1)}{\partial X}\right) = 0, \tag{8}$$

$$r(\tau, X, I_1) + \lambda \tilde{r}(\tau, X, I_1) - \overline{\kappa}_I + \underline{\kappa}_I = 0, \tag{9}$$

$$\lambda[\theta R(I_1, E^a) - \tau E(X, I_1) + \psi T + \mu(I_0 - I_1) - C(X, I_1)] = 0. \tag{10}$$

The first order condition with respect to X in Eq. (8) shows that borrowers choose abatement trading off a reduction in the emissions tax bill against the cost of abatement. Eq. (9) is the first order condition with respect to I_1, and it reflects the trade-off between increasing the private net return and relaxing the financial constraints, captured by $r(\cdot)$ and $\lambda \tilde{r}(\cdot)$ respectively. Together with Eq. (10), which combines the complementary slackness conditions of the financial constraint (5) and non-negativity constraint of c_1 defined in (6), these conditions define the optimal I_1, X, and λ for a given τ.

Lemma 1. *Borrowers liquidate investment only if the financial constraint (5) binds: $I_1 < I_0$ only if $\lambda > 0$.*

Proof. See Appendix A.3 □

Lemma 1 follows from Assumption 2, which implies that the net marginal return is positive and therefore it is optimal to continue the project without any liquidations, i.e., the optimum is a corner solution with $I_1 = I_0$. By contrast, if the financial constraint is binding, the pledgeable income under the full investment scale is insufficient to support the required borrowing. Since liquidations relax financial constraints (by Assumption 2.2), in this case borrowers reduce the investment scale at $t = 1$ by choosing $I_1 < I_0$.

Definition 2. Given an emissions tax τ, the competitive equilibrium is the set of allocations $I_1^*(\tau), X^*(\tau), \lambda^*(\tau)$, defined by Eqs. (8), (9) and (10). Aggregate emissions are given by $E^a(\tau) = E(X^*(\tau), I_1^*(\tau))$. The allocations $c_1^*(\tau)$ and $c_2^*(\tau)$ follow from the respective budget constraints.

For brevity we sometimes omit the dependence of equilibrium allocations on τ. For instance, we refer to $X^*(\tau)$ as X^* or to $I_1^*(\tau)$ as I^*.

3.3. Pigouvian benchmark

Proposition 2. *If $\lambda^*(\gamma) = 0$, then the competitive equilibrium with $\tau = \gamma$ is equivalent to the first-best allocation.*

Proof. With $\lambda^*(\gamma) = 0$, it follows from Lemma 1 that $I_1^* = I_0$. This investment level, as well as the FOC of borrowers w.r.t. X in Eq. (8), are then equivalent to those in the first best given in Proposition 1. □

Proposition 2 establishes an important benchmark result. If the financial constraint is slack, then by Lemma 1 borrowers can avoid inefficient liquidations, and the optimal Pigouvian emissions tax can implement the first-best allocation. Accordingly, throughout we refer to a tax $\tau = \gamma$ as the *Pigouvian benchmark*. Note that this is a general benchmark which could also implement the first-best allocation if borrowers had heterogeneous cost functions (see Internet Appendix IA.3). In the next section we analyze optimal emissions taxes when financial constraints bind and explore how they depart from this benchmark.

4. Optimal carbon pricing

This section analyzes optimal emissions taxes in the presence of financial constraints. We then show under what conditions the resulting equilibrium allocation is constrained efficient, and ask whether there is a case to use other policy instruments.

4.1. Socially optimal emissions tax

We solve the problem of a regulator choosing the optimal emissions tax τ^* at $t = 1$ so as to maximize social welfare. This problem is formally stated in Appendix B.3. The regulator's first order condition with respect to τ can be written as

$$(\tau - \gamma)\frac{\partial E(X, I_1)}{\partial X}\frac{\partial X^*}{\partial \tau} + r(\gamma, X^*, I_1^*)\frac{\partial I_1^*}{\partial \tau} + \kappa_\tau = 0, \tag{11}$$

where κ_τ is the Lagrange multiplier on the non-negativity constraint $\tau \geq 0$.

The regulator trades off the effect of the tax on welfare through its impact on emissions, reflected in the first term in Eq. (11), against the welfare implications of the change in the final investment scale induced by the tax, captured in the second term of the equation. In this condition, the final investment scale I_1^* and abatement X^* are optimal choices by borrowers that respond to changes in emissions taxes. We next discuss how emissions taxes affect borrowers' choices and then use these insights to characterize the optimal emissions tax.

4.1.1. The effect of taxes on equilibrium allocations

Higher emissions taxes increase the cost of polluting, which incentivizes borrowers to invest more in abatement. But higher emissions taxes also affect the tightness of financial constraints, which may induce borrowers to abate less. Through this indirect effect, emissions taxes can have a perverse effect and decrease abatement due to tightening financial constraints. In this case, emissions taxes would be counterproductive and not a useful tool to reduce emissions in the first place. We discuss this case in Appendix B.1. In the main text, we focus on the interesting case in which emissions taxes are a useful tool to incentivize abatement, so that $\partial X^*/\partial\tau > 0\ \forall\tau$ (in the Appendix we also characterize sufficient conditions for this to hold).

The following Lemma additionally clarifies how liquidations and therefore the equilibrium investment scale I_1^* responds to emissions taxes.

Lemma 2. *If the financial constraint is slack, then emissions taxes do not affect the final investment scale, $\partial I_1^*/\partial\tau = 0$. If the financial constraint binds, then there exists a threshold $\hat{\gamma}^p(\tau)$ such that*

- *if $\gamma^p < \hat{\gamma}^p(\tau)$, then emissions taxes reduce the final investment scale $\partial I_1^*/\partial\tau < 0$;*
- *if $\gamma^p = \hat{\gamma}^p(\tau)$, then emissions taxes do not affect the final investment scale $\partial I_1^*/\partial\tau = 0$;*
- *if $\gamma^p > \hat{\gamma}^p(\tau)$, then emissions taxes increase the final investment scale $\partial I_1^*/\partial\tau > 0$.*

Proof. See Appendix B.2 □

Only if the financial constraint binds, borrowers need to liquidate investments. Interestingly, higher emissions taxes can result in more or less liquidations, depending on how strongly payoffs are affected by physical climate impacts captured by $\gamma^p = -\partial R(I_1, E^a)/\partial E^a$. To dissect the result in Lemma 2, the overall effect of emissions taxes on the final investment scale can be derived from totally differentiating (10) with respect to τ:

$$\frac{\partial I_1^*}{\partial\tau} = \frac{(1-\psi)E(X^*, I_1^*) - \psi\tau\frac{\partial E^a}{\partial X}\frac{\partial X^*}{\partial\tau} + \theta\gamma^p\frac{\partial E^a}{\partial X}\frac{\partial X^*}{\partial\tau}}{\tilde{r}(\tau(1-\psi) + \theta\gamma^p, X^*, I_1^*)} \tag{12}$$

This equation highlights the three channels through which financial constraints shape the effect of emissions taxes on the final investment scale. First, changes in the tax directly affect the size of the tax bill and the tax rebate. Since only a fraction ψ of the tax rebate is pledgeable, this *direct* negative effect of the emissions tax on the tightness of the financial constraint is captured by the term $(1-\psi)E(X^*, I_1^*)$. Second, higher taxes incentivize more abatement, which lowers aggregate emissions and reduces the pledgeable tax rebate $\psi T = \psi\tau E^a$. This is captured by the second term in the numerator of (12).[4] Through both the direct effect and the tax rebate effect higher taxes deplete pledgeable income, resulting in a lower final investment scale.

The third channel operates in the opposite direction and implies that higher emissions taxes partly *relax* financial constraints. Higher emissions taxes result in a lower aggregate level of emissions. This has a positive impact on borrowers' pledgeable resources, because it reduces future physical climate impacts on borrowers' assets. This channel represents a collateral externality of emissions: each unit of prevented aggregate emissions increases borrowers' pledgeable resources by $\theta\gamma^p$, reflected in the third term in the numerator of (12).

[4] The total effect of abatement on pledgeable income also includes changes in abatement costs and is given by $-\left[\partial C(X, I_1)/\partial X + \tau\partial E(X, I_1)/\partial X - \psi\tau\partial E^a/\partial X\right]\partial X^*/\partial\tau$. However, using the borrower's optimal abatement choice in Eq. (8), this term simplifies to $\psi\tau\partial E^a/\partial X \times \partial X^*/\partial\tau$ in Eq. (12).

Overall, the effect of emissions taxes on financial constraints and liquidations depends on the relative strength of the collateral externality. When borrowers' exposure to physical climate impacts is low such that $\gamma^p < \hat{\gamma}^p$, higher emissions taxes imply tighter constraints and more liquidations. By contrast, if $\gamma^p > \hat{\gamma}^p$, the equilibrium effect of the collateral externality dominates, so that higher emissions taxes relax financial constraints and result in fewer liquidations.

4.1.2. Optimal emissions tax

Because emissions taxes interact with financial constraints, the regulator considers not only the direct effect of taxes on emissions, but also their side effect on asset liquidations.

Proposition 3. *The optimal emissions tax τ^* solves (11). If the financial constraint is slack, then $\tau^* = \gamma$. If the financial constraint binds, then*

- *$\tau^* < \gamma$ if $\gamma^p < \hat{\gamma}^p(\tau^*)$,*
- *$\tau^* = \gamma$ if $\gamma^p = \hat{\gamma}^p(\tau^*)$,*
- *$\tau^* > \gamma$ if $\gamma^p > \hat{\gamma}^p(\tau^*)$,*

where the threshold $\hat{\gamma}^p(\tau^*)$ is defined in Lemma 2 and Appendix B.2.

Proof. See Appendix B.3 □

With binding financial constraints, the optimal emissions tax generally differs from the Pigouvian benchmark equal to the direct social cost of emissions γ. We stress that $\gamma = 2\gamma^u + \gamma^p$ already includes both the utility component γ^u as well as the physical climate damage component γ^p. Therefore, Proposition 3 implies that the optimal tax may be $\tau^* > 2\gamma^u + \gamma^p$ if γ^p is sufficiently large.

To understand this result, first suppose aggregate emissions do not affect borrowers' payoffs, i.e., $\gamma^p = 0$ and $\gamma = 2\gamma^u$. In this case, binding financial constraints unambiguously imply an optimal emissions tax $\tau^* < \gamma$. The reason is that with $\gamma^p = 0$, financial constraints do not result in a collateral externality, and emissions taxes affect financial constraints only through the negative direct and tax rebate effects on pledgeable income. As a result, in this case higher taxes unambiguously trigger more inefficient liquidations by Lemma 2. Internalizing this undesired side effect, an environmental regulator sets an emissions tax below the direct social cost of emissions, $\tau^* < \gamma$.

However, as discussed above, with $\gamma^p > 0$ physical climate impacts imply that emissions taxes also affect borrowers' financial constraints through a *collateral externality*. This collateral externality works in the opposite direction, and if $\gamma^p > \hat{\gamma}^p$, then the effect of aggregate emissions on pledgeable income is sufficiently high such that the collateral externality dominates the negative effects of the tax. In this case, higher emissions taxes *relax* financial constraints by Lemma 2. This fundamentally changes the trade-offs faced by an environmental regulator, implying optimal emissions taxes above the direct social cost of emissions, $\tau^* > \gamma$. Conversely, if $\gamma^p < \hat{\gamma}^p$, as in the example with $\gamma^p = 0$, the negative direct and tax rebate effects dominate the collateral externality, and the optimal tax is $\tau^* < \gamma$.

Thus, the collateral externality of emissions warrants higher optimal emission taxes if emissions disproportionately affect the pledgeable resources of financially constrained agents. For example, the optimal tax may be above the Pigouvian benchmark if the value of collateralizable assets — such as real estate or productive equipment — are highly exposed to climate risks like droughts, floods, or heat waves, which become more likely when emissions remain high. Conversely, if the social costs of emissions primarily affect assets with limited collateral value, such as natural ecosystems or human capital, the collateral externality is weak. In such a case, financial constraints call for an optimal emissions tax below the Pigouvian benchmark.

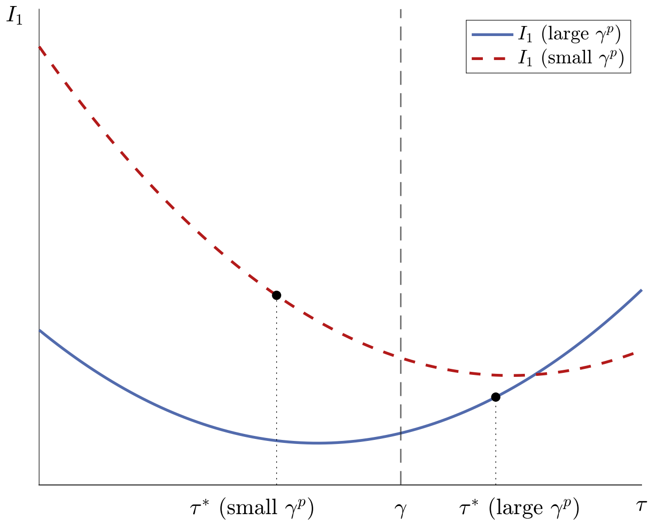

Fig. 1. Final investment scale and optimal emissions taxes.
This figure uses a numerical example to plot the equilibrium final investment scale I_1 as a function of the emissions tax τ for a large and a small value of physical climate impacts γ^p, holding the overall social cost of emissions γ constant. Details on the numerical solution are provided in the Internet Appendix (Section IA.8).

Numerical example. To illustrate the results in Proposition 3 and Lemma 2, we solve the model numerically using the functional forms $R(I_1, E^a) = \rho I_1 - \gamma^p E^a$, $C(X, I_1) = \eta X^2 I_1/2$, and $E(X, I_1) = max\left\{(1-\beta X)I_1, 0\right\}$, where η and β are parameters governing the cost and benefit of abatement, respectively. Details on the parametrization, derivations, and numerical solution are provided in the Internet Appendix (Section IA.8).

Using these functional forms, the borrower's first order condition (8) implies an interior solution $X = \tau\beta/\eta$. The optimal I_1 is plotted as a function of the emissions tax τ for two separate cases in Fig. 1. In the first case (red dashed line), we set γ^p to a relatively small number and γ^u to a relatively large number. Conversely, in the second case (blue solid line) γ^p is relatively large and γ^u relatively small. Importantly, we hold the overall social cost of emissions γ constant across the two cases, so that only the *relative* weight of physical climate impacts γ^p varies.

We compute the optimal emissions tax for both cases and find that it is below the Pigouvian benchmark rate γ if γ^p is relatively small, but above γ if γ^p is relatively large — illustrating Proposition 3. Moreover, in line with Lemma 2, the optimal emissions tax is at a level where the final investment scale I_1 decreases in the emissions tax if γ^p is small, but increases if γ^p is relatively large. Since we are holding the overall cost of emissions γ constant, this example illustrates how the predominant type of costs of emissions matters for the optimal level of emissions taxes. If a sufficiently high share of the costs of aggregate emissions affects pledgeable assets (i.e. if γ^p is high relative to γ^u), the climate-induced collateral externality is strong enough to motivate an optimal emissions tax above the Pigouvain benchmark.

Collateral externality of emissions. Previous literature on collateral externalities focuses primarily on pecuniary externalities, whereby borrowers do not internalize how their choices affect financial constraints through their impact on fire sale discounts (for a detailed discussion, see Dávila and Korinek, 2018). By contrast, in our setting collateral externalities can emerge because agents do not internalize their impact on financial constraints through *aggregate emissions*. Proposition 3 shows that this climate-induced collateral externality may imply optimal emissions taxes above a standard Pigouvian benchmark. The more general point is that, even if the collateral externality is not strong enough to motivate taxes above a standard Pigouvian benchmark, it shapes optimal emissions taxes as it dampens the negative effect of taxes on financial constraints. In the next subsection, we demonstrate this further by deriving a generalized Pigouvian benchmark tax that accounts for climate-induced collateral externalities.

4.2. Efficiency and generalized pigouvian benchmark

In the presence of financial constraints, the total social cost of emissions includes not only the direct social cost of emissions $\gamma = 2\gamma^u + \gamma^p$, but also the indirect cost due to collateral externalities driven by physical climate impacts. We next define a generalized Pigouvian benchmark that accounts for this collateral externality. This enables us to demonstrate how the optimal tax is shaped by the interaction between financial constraints and physical climate impacts, and evaluate the efficiency of the allocation that it implements. Specifically, we define the generalized Pigouvian benchmark as the emissions tax that equalizes the private cost of emissions τ to the *total* social cost of emissions. The total cost accounts for all marginal effects of increasing aggregate emissions in the presence of financial constraints. It thus sums the direct cost γ and the shadow cost of the climate-induced collateral externality $\lambda\theta\gamma^p$. Additionally, the total social cost accounts for the fact that a fraction ψ of tax rebates are pledgeable, generating a marginal benefit $\lambda\psi\tau$.

Definition 3. The generalized Pigouvian tax is given by

$$\tau^{GP} = \frac{\gamma + \lambda^*\theta\gamma^p}{1 + \psi\lambda^*}. \tag{13}$$

Financial constraints motivate a generalized Pigouvian benchmark tax τ^{GP} that takes into account the interaction of aggregate emissions with financial constraints. The generalized Pigouvian tax τ^{GP} increases in borrowers' exposure to physical climate impacts γ^p through two channels. First, higher γ^p mechanically increases the direct costs of emissions γ. Second, larger physical climate impacts increase the magnitude of the collateral externality captured in $\lambda^*\theta\gamma^p$. This latter channel is stronger when financial constraints of borrowers are tighter, as reflected by the cross derivative $\partial^2\tau^{GP}/(\partial\gamma^p\partial\lambda^*) = \theta/(1+\psi\lambda^*)^2 > 0$. This highlights that the interaction between physical climate impacts and financial constraints is central to why the collateral externality of emissions drives up the generalized Pigouvian benchmark tax. Finally, note that if financial constraints are slack, then the generalized Pigouvian tax coincides with the standard Pigouvian benchmark: $\tau^{GP} = \gamma$ if $\lambda^* = 0$. The next subsection compares this generalized benchmark to the optimal emissions tax derived in Proposition 3 and evaluates the efficiency of the allocation implemented with emissions taxes.

4.2.1. Are carbon taxes enough?

With binding financial constraints, the optimal emissions tax differs from a standard Pigouvian benchmark and cannot implement the first-best allocation. One may expect this implies that it is beneficial to complement or replace emissions taxes with other policy tools. However, this does not need to be the case because the optimal emissions tax may still implement the second-best, constrained-efficient allocation. In such circumstances, there is no benefit to introducing other policy tools.

In the constrained-efficient allocation, which is formally defined in Appendix C, a social planner can choose X and I_1 directly, subject to the same resource and financial constraints as private agents. This contrasts with the regulator's problem, where abatement is incentivized through an emissions tax. The following proposition compares the optimal emissions tax τ^* to the generalized Pigouvian benchmark τ^{GP} and evaluates whether emissions taxes alone can implement the second best.

Proposition 4. *If the financial constraint binds and*

- $\psi = 1$*, then* $\tau^* = \tau^{GP}$ *and the competitive equilibrium is constrained efficient,*

- *$\psi < 1$, then $\tau^* < \tau^{GP}$ and the competitive equilibrium is not constrained efficient.*

Proof. See Appendix C □

In contrast to a social planner, the environmental regulator cannot choose abatement directly, but instead uses emissions taxes as a policy instrument to incentivize abatement. If tax rebates are fully pledgeable, the regulator can implement the constrained-efficient abatement level without introducing additional distortions to the final investment scale by setting the emissions tax equal to the generalized Pigouvian benchmark tax τ^{GP} from Definition 3. In contrast, if tax rebates are not fully pledgeable, $\psi < 1$, taxes have a direct adverse effect on financial constraints because $\tau E(X, I_1) - \psi T > 0$, and the regulator needs to set an emissions tax below τ^{GP}. As a result, emissions taxes can only implement the constrained-efficient allocation if tax rebates are fully pledgeable.

This result implies that, when $\psi < 1$, there may be scope to improve welfare by using policy tools other than carbon taxes. The following section discusses a cap-and-trade system with tradable pollution permits in this context.

4.3. Cap and trade

An alternative policy tool that can curb emissions is a cap-and-trade system with a limited quantity Q of tradable pollution permits (similar to the EU ETS). Absent other frictions, such pollution permit markets are equivalent to emissions taxes, and the Coase Theorem implies that the initial allocation of pollution permits does not affect the equilibrium level of emissions (see Coase, 1960; Montgomery, 1972). In what follows we show that this is not necessarily the case in the presence of financial constraints, and explore whether a cap-and-trade system can achieve higher welfare than emissions taxes.

For each unit of emissions, a borrower needs to surrender a permit to the regulator at $t = 2$. We assume that a share ϕ of all permits Q is freely allocated to borrowers ex-ante, and that the remaining $(1 - \phi)Q$ permits need to be purchased by the borrower at the market price p. To simplify the exposition, we assume here that the proceeds from permit sales accrue to investors (Internet Appendix Section IA.2.3 shows that the insights derived here also hold if instead the proceeds accrue to borrowers). Borrowers can trade permits with each other at the market price p. Note that with freely allocated permits borrowers retain the same incentives to invest in abatement because of the opportunity cost of selling unused permits.

4.3.1. Mapping cap-and-trade to emissions taxes

The budget constraints of the borrower and the first order conditions under the cap-and-trade system are stated in Appendix D. The FOCs are equivalent to those in the baseline problem, with p taking the place of τ. The borrower's FOC with respect to abatement determines the relationship between the privately optimal level of abatement X and the permit price p, and mirrors Eq. (8) of the original problem:

$$(1 + \lambda)\left(p\frac{\partial E(X, I_1)}{\partial X} + \frac{\partial C(X, I_1)}{\partial X}\right) = 0, \tag{14}$$

This condition, together with the market clearing for permits, $Q = E^a$, jointly determine a mapping from p to E^a. Consequently, we can express the regulator's problem as maximizing social welfare by choosing p. Appendix D reports the first order condition of the regulator. As in the baseline setting, the regulator internalizes the effect of the policy on borrowers' profits and emissions. Comparing the FOCs under the cap-and-trade system with the one in the original problem yields the following result.

Proposition 5. *The equilibrium allocation implemented with a cap-and-trade system is equivalent to the one implemented with emission taxes if $p = \tau$ and $\phi = \psi$.*

A cap-and-trade system that allocates all permits for free, $\phi = 1$, and implements a permit price $p^* = \tau^{GP}$, results in a constrained-efficient allocation.

Proof. See Appendix D □

In both the baseline setting with emissions taxes and the cap-and-trade system the regulator's policy amounts to choosing the private marginal cost of emissions represented either by the tax rate τ or the permit price p. The direct effect of the policies on the financial constraints depend, respectively, on the pledgeability of the tax rebates ψ, and the share of freely allocated permits ϕ. Pollution permits have a direct effect on the financial constraint if borrowers need to purchase some of them ex-ante (i.e. if $1 - \phi > 0$). This corresponds to the direct effect of the tax bill on pledgeable income under emissions taxes. The price of permits also affects the tightness of the financial constraints indirectly, mirroring the effects of emissions taxes discussed in Section 4.1.2 (including the collateral externality).

The advantage of using a cap-and-trade system instead of emissions taxes is that the regulator can choose ϕ optimally. The equivalence result in Proposition 5 implies that the regulator can avoid the problem of the carbon price's direct effect on borrowers' financial constraints altogether by allocating all permits for free, i.e. setting $\phi = 1$. In this case, the shadow cost of permits induces borrowers to engage in a constrained-efficient level of abatement, as stated in the second part of Proposition 5.

4.3.2. Coasean independence

An implication of the Coase Theorem is that absent other frictions the initial allocation of the pollution allowances does not affect the equilibrium level of externality (see Montgomery, 1972). This result is often referred to as the independence property in the literature. Proposition 5 combined with our previous results show that independence does not hold under financial frictions. This is consistent with recent empirical evidence from the EU ETS that indicates that independence holds for large emitters but not for smaller firms (see Zaklan, 2023), as small firms are more likely to be financially constrained.

Our result also complements previous findings that show independence fails under frictions such as transaction costs (Stavins, 1995) and market power (Hahn, 1984; Fowlie et al., 2016). These contributions show that allocating more permits for free can lead to higher emissions if marginal transaction costs are decreasing or incumbents have market power. Our next result relates to these findings by showing that, in the presence of financial constraints, allocating more permits at no cost may instead result in lower emissions.

Lemma 3. *An increase in the number of freely allocated permits results in lower aggregate emissions at a given permit price p if and only if*

$$\frac{\partial E(X, I_1)}{\partial X}\frac{\partial X^*}{\partial I_1} + \frac{\partial E(X, I_1)}{\partial I_1} < 0. \tag{15}$$

Proof. See Appendix D □

Allocating more permits for free limits the direct impact of the cap-and-trade system on borrowers' pledgeable income. This can increase or decrease aggregate emissions, depending on whether borrowers respond to laxer financial constraints predominantly by liquidating less or by abating more. As we show in the appendix, if the condition in Lemma 3 holds, then relaxing financial constraints results in lower emissions because the underlying technologies are such that abatement is more efficient at a higher investment scale.[5] In this case, allocating more permits for free induces a reduction in aggregate emissions for a given permit price. Vice versa, if the condition in Lemma 3 does not hold, increasing the number of freely allocated permits leads to a higher level of aggregate emissions because the effect of relaxing financial constraints on the final investment scale dominates.

4.4. Policy implications

Propositions 4 and 5 imply that — despite the presence of binding financial constraints — a regulator can implement a constrained-efficient allocation by combining carbon taxes with a fully pledgeable tax rebate, or by using a cap-and-trade system with freely allocated permits. An important policy implication is that a pollution permit market with free allowances may be a superior policy instrument to carbon taxes in the presence of financial constraints and partially pledgeable tax rebates.

Yet, in practice cap-and-trade systems often do not allocate permits for free. For example, the EU ETS (the largest emissions permit market in the world), only grants free allowances equal to a fraction of total emissions, and is gradually reducing the amount of free allowances over time. For example, the manufacturing industry received 80% of its allowances for free in 2013. This proportion had been decreased down to 30% in 2020 (European Commission, 2024).

We note that there may be considerations outside our model that motivate these real-life policy choices. For example, it may be difficult for regulators to correctly allocate free permits if polluters were privately informed about heterogeneous abatement costs, potentially triggering undesirable distributional consequences. Determining the amount of freely allocated permits by past emissions (a policy referred to as "grandfathering") could help alleviate this issue, but may weaken incentives to reduce emissions as firms may aim to avoid a reduction in the amount of freely allocated permits in the future (see Clò, 2010). Additionally, under heterogeneity it may be beneficial to use non-linear emissions taxes to ensure different polluters face the private cost of emissions that more accurately accounts for their individual constraints (see Hoffmann et al., 2017). In the Internet Appendix (Section IA.3), we discuss how this may call for combining emissions taxes and cap-and-trade to obtain the distinct benefits of these two policy tools: emissions taxes can be set non-linearly, while cap-and-trade enables the regulator to allocate permits for free, limiting the negative effect due to non-pledgeable tax rebates. While modeling all these frictions is beyond the scope of this paper, our results highlight that regulators should also weigh the adverse impact of allowance sales on the tightness of financial constraints when accounting for these additional forces.

Regulatory pecking order. Our results also imply that financial constraints do not necessarily motivate targeting climate-related objectives using other policy tools, such as financial regulatory tools or monetary policy. In fact, in the model there is a clear "regulatory pecking order" whereby regulators should first design carbon pricing in a way that minimizes the adverse effect on financial constraints before resorting to other policy tools. While the precise conditions for constrained efficiency may not be satisfied in real life, at a minimum, our results highlight the importance of prioritizing limiting the impact of carbon pricing on pledgeable income.

In the next section we extend the baseline model to allow for an initial leverage choice and show that these insights carry over also to this setting. In the Internet Appendix (Section IA.2), we also analyze a quantity limit on pollution, non-linear taxes, and green subsidies. These policies may implement the constrained-efficient allocation if designed appropriately, but can have limitations that we discuss in the Internet Appendix.

5. Extensions

5.1. Initial leverage choice and regulation

In the baseline model, borrowers have no initial leverage at $t = 0$, yet in reality the tightness of financial constraints is often affected by debt raised in the past. Motivated by this observation and by recent debates on whether financial regulation should include climate-related goals (for example, see Brunnermeier and Landau, 2021), this section extends the model by allowing for the tightness of $t = 1$ financial constraints to be influenced by a borrower's previous capital structure choices. In particular, we introduce the ex-ante date $t = 0$ during which borrowers decide how to finance their project. This allows us to explore how borrowers' financing choices interact with carbon pricing.

5.1.1. Additional model ingredients

At $t = 0$ each borrower receives a limited endowment A^b and has access to a fixed-scale project requiring an upfront investment of I_0. Borrowers can fund the project with a mix of inside equity $e \le A^b$ and external debt financing $d_0 = I_0 - e$. We do not restrict d_0 to be positive (negative d_0 can be interpreted as cash holdings). We assume that borrowers issue short-term debt due at $t = 1$, at which stage they can issue d_1 as in the baseline model.[6] If borrowers fail to repay the debt at $t = 1$, creditors can force a (partial) liquidation of the investment and appropriate all of the liquidation proceeds. We show in the appendix that this implies there is no default on $t = 0$ debt at $t = 1$ and that the financial constraints at $t = 0$ are slack whenever borrowers prefer to start, rather than forgo, the project. To introduce a meaningful trade-off for borrowers in how much inside equity they contribute to the project, we assume that borrowers' consumption utility at $t = 0$, $u(c_0)$, is concave, $u'(c_0) > 0$, $u''(c_0) < 0$. To ensure an interior solution we assume it also satisfies $u'(0) = \infty$ and $u'(\infty) = 0$. Hence, borrowers' overall utility (1) is replaced by

$$U^b = u(c_0) + c_1 + c_2 - \gamma^u E^a, \tag{16}$$

Focusing on the relevant case in which borrowers prefer to start the project, their problem is to maximize utility subject to (4), (5), (6), (7), as well as an additional $t = 0$ budget constraint and a revised $t = 1$ budget constraint stated in Appendix E. We close the model by assuming that investors remain risk-neutral through all periods and receive a large endowment A^i at both $t = 0$ and $t = 1$.

5.1.2. Borrowers' optimal choices

The $t = 1$ problem of the borrower is similar to the baseline model, with the FOCs with respect to abatement and investment scale given by (8) and (9). The only difference is that the complementary slackness condition (10) is replaced by a condition that also depends on $t = 0$ debt:

$$\lambda[\theta R(I_1, E^a) - \tau E(X, I_1) + \psi T + \mu(I_0 - I_1) - C(X, I_1) - d_0] = 0. \tag{17}$$

Liquidations at $t = 1$ still depend on whether the financial constraint binds or not, as described in Lemma 1. Critically, with $t = 0$ debt the financial slack available to borrowers at $t = 1$ is endogenous to their initial leverage.

[5] We show in Appendix D that the sign of $\partial X^*/\partial I_1$, i.e. whether abatement is more efficient at higher investment scale, depends on the underlying production technologies. This may differ by industry. For example, consistent with $\partial X^*/\partial I_1 > 0$, evidence from (Martinsson et al., 2024b) shows that larger firms are more efficient at reducing emissions following a shock to emissions prices in the Swedish trucking industry, and Bellon and Boualam (2024) find that financially distressed firms in the US increase their pollution intensity.

[6] Since there is no risk, this is equivalent to assuming borrowers can issue long-term debt and have access to a storage technology. In this case the total amount of debt due at $t = 2$ would be subject to a constraint as in (5) and the problem would be equivalent to the one studied here.

At $t = 0$ borrowers decide on their capital structure by choosing the optimal inside equity contribution e. Outside debt financing follows as the residual from $d_0 = I_0 - e$. The first order condition of the borrower's problem w.r.t. e is given by

$$u'(A^b - e) - 1 = \lambda. \tag{18}$$

Thus, borrowers contribute equity trading off its effect on marginal utilities of consumption at $t = 0$ and $t = 1$ against the benefit of relaxing financial constraints, captured by λ.

Condition (18) implies that, if the borrower's endowment A^b is sufficiently low, the high marginal utility of $t = 0$ consumption justifies choosing high initial leverage even if that means future financial constraints will bind. In what follows we focus on this case.

5.1.3. Carbon pricing and socially optimal leverage

We now revisit optimal carbon pricing and ask whether borrowers' leverage choices are socially efficient. To that end, we extend the regulatory toolbox and allow the regulator to complement the emissions tax with a $t = 0$ leverage mandate that fixes the borrower's inside equity at a level $\bar{e}$. Such a policy could be implemented through a direct mandate or taxes and subsidies on external financing (see Internet Appendix IA.2.4).[7] To remain close to the baseline model, we assume that the regulator sets the emissions tax at $t = 1$ (rather than committing to a tax at $t = 0$).

We show in Appendix E that the optimal emissions tax in the extended setting is still characterized by Proposition 3. This is because borrowers' choice of d_0 does not change the nature of the trade-off faced by the regulator between incentivizing abatement and triggering inefficient liquidations for a *given* tightness of the financial constraints at $t = 1$.

The optimal leverage regulation affects a different decision margin: it changes the extent to which the borrower uses $t = 0$ resources to consume vs to relax the $t = 1$ financial constraints, as can be seen from the regulator's first order condition w.r.t. $\bar{e}$:

$$u'(A^b - \bar{e}) - 1 = \left[r(\gamma, X^*, I_1^*) - (\gamma - \tau^*) \frac{\partial E(X, I_1)}{\partial X} \frac{\partial X^*}{\partial I_1} \right] \frac{\partial I_1^*}{\partial \bar{e}}. \tag{19}$$

The first order condition trades off the marginal utility of consumption against the marginal social value of more financial slack at $t = 1$. The latter consists of the value due to a higher net return on the project (captured by $r(\gamma, X^*, I_1^*)$ in Eq. (19)), and the value due to the change in abatement and its effect on aggregate emissions (captured by the remaining terms in the square brackets). Notice that, by the envelope theorem, the regulator does not consider the effect of the leverage mandate on the emissions tax because the tax is set optimally at $t = 1$.

Comparing (18) and (19) implies that whether the private choice of leverage is socially optimal depends on whether the private value of financial slack coincides with the social value (as reflect in the right-hand side of the respective equations). While both the borrower and the regulator account for the effect of equity on the pecuniary return generated by the project, the social and private value of emissions may differ.

Proposition 6. *If the financial constraint binds at $t = 1$, the borrower's choice of equity*

- *is socially optimal whenever $\psi = 1$,*
- *is not socially optimal whenever $\psi < 1$ and $dE(X^*, I_1^*)/d\bar{e} \neq 0$.*

Proof. See Appendix E □

When tax rebates are fully pledgeable, then the regulator sets a tax equal to the generalized Pigouvian benchmark τ^{GP}, as in the baseline model. This implies that emissions are priced correctly, which ensures that the private and social value of emissions coincides. Hence, the marginal private and social values of financial slack are equalized, so that the choice of leverage by borrowers is socially optimal when $\psi = 1$. By contrast, if $\psi < 1$ the optimal emissions tax is below the generalized Pigouvian benchmark, so that there is a wedge between the social and private cost of emissions. As a result, in this case leverage regulation can improve welfare.

Appendix E shows that, if $\psi < 1$, the socially optimal equity can be above or below the privately optimal level depending on how equity affects emissions. Higher borrower equity loosens financial constraints. This can affect emissions in two ways. On one hand, it implies more emissions due to a higher final investment scale. On the other hand, looser financial constraints affect the optimal abatement choice, which may lower emissions. Whether the effect on abatement dominates depends on the cross-derivatives of the emissions and abatement functions (it requires abatement to be more efficient at a higher investment scale).

If the effect of equity on abatement dominates, such that $dE(X^*, I_1^*)/d\bar{e} < 0$, then the socially optimal equity is above the privately optimal level, $\bar{e}^* > e^*$. By contrast, if $dE(X^*, I_1^*)/d\bar{e} > 0$, then higher equity implies higher emissions, and the socially optimal equity level is below a borrower's optimal choice, $\bar{e}^* < e^*$. Note that $\bar{e}^* < e^*$ may be socially optimal even though liquidations are inefficient. This is because the emissions tax is already set to optimally distribute the $t = 1$ resources between abatement and avoiding liquidations, while the role of the leverage mandate is to correct the consumption choice that may not fully internalize the social benefits of higher leverage reducing emissions. This result mirrors insights in Dávila and Walther (2022) that, with constraints on the regulation of some externality-generating activity (here abatement), the optimal second-best regulation of other choices (here leverage) depends on Pigouvian wedges in the constrained regulation and on how the perfectly regulated choices affect the imperfectly regulated activity. In Appendix E.4 we discuss in detail how emissions and abatement technologies shape these wedges and determine the direction of the optimal leverage regulation in our framework.

To sum up, if the regulator can design environmental policy that avoids directly impacting financial constraints (e.g. by using fully pledgeable tax rebates or a cap-and-trade system with freely allocated permits), then there is no need to complement it with leverage regulation. Otherwise, leverage regulation can improve welfare by ensuring that the choice of funding mix internalizes the social benefit of reducing emissions through leverage. A difficulty in using this tool is that the optimal policy may need to constrain or limit leverage, depending on the specifics of the underlying technologies that link abatement and investment scale to costs and emissions.

5.2. Financial markets

This section analyzes how financial markets interact with emissions taxes. First, we introduce socially responsible investors (SRIs). We then introduce risk to the model to study the role of hedging, long-term debt and external equity financing.

5.2.1. Socially responsible investing

We assume that SRIs derive utility ω from emissions reductions by the borrowers they provide funding to at $t = 1$, relative to a benchmark $\tilde{E}$ (see Internet Appendix IA.4 for a formal statement of investors' preferences). SRIs' break even requires that $d_1 = r_1 d_1 + \omega[\tilde{E} - E(X, I_1)]$, where r_1 is the gross interest rate charged by SRIs. The case $\tilde{E} = 0$ corresponds to SRIs receiving negative utility from any emissions generated by borrowers, whereas $\tilde{E} > 0$ represents impact investors who receive positive utility if a borrower reduces emissions sufficiently.

[7] Such policies could be applied directly to non-financial firms, or introduced into the Basel regulatory framework if borrowers are interpreted as financial institutions. We offer such reinterpretation of our framework in Internet Appendix IA.6 where we assume borrowers' projects represent lending to firms with polluting assets by financial institutions.

For SRIs to have an impact, it must be that borrowers cannot easily substitute away from SRIs to purely financially-motivated investors. For simplicity, we assume here that all investors are socially responsible, so that borrowers cannot substitute SRI capital for cheaper financial capital (for a discussion on SRIs achieving impact even when purely financially-motivated capital is abundant, see Oehmke and Opp, 2025).

Under these assumptions, the borrower's problem now yields the following FOC for abatement and complementary slackness condition:

$$(1+\lambda)\left[(\tau+\omega)\frac{\partial E(X,I_1)}{\partial X}+\frac{\partial C(X,I_1)}{\partial X}\right]=0, \tag{20}$$

$$\lambda[\theta R(I_1,E^a)-\tau E(X,I_1)+\omega[\tilde{E}-E(X,I_1)]+\psi T-d_0 + \mu(I_0-I_1)-C(X,I_1)]=0. \tag{21}$$

These correspond to the original conditions (8) and (17), with $\tau+\omega$ taking the place of τ in (20), and with the new term $\omega[\tilde{E}-E(X,I_1)]$ showing up in the complementary slackness condition (21). Eq. (20) captures the incentive effect of SRIs on abatement due to the financing cost being proportional to emissions reductions. This incentive effect works in the same way as an emissions tax. A critical difference between the tax and the SRI premium is the effect on financial constraints, as seen in the complementary slackness condition (21). The term $\omega[\tilde{E}-E(X,I_1)]$ can be positive if firms reduce emissions below the target level $\tilde{E}$. In this case, SRIs relax financial constraints. However, if the term is negative, the disutility SRIs derive from lending to polluters tightens financial constraints. This contrasts with the effect of the emissions tax on the financial constraint, which is (partially) offset by the pledgeable tax rebate and thus equal to $\psi T-\tau E(X,I_1)$.

Corollary 1. *Suppose investors derive utility ω from emissions reductions relative to a benchmark $\tilde{E}$ by borrowers they provide financing to. Then, abatement and liquidations with $\tau=0$ are equivalent to those in the baseline model without socially responsible investors but with an emissions tax $\tau=\omega$ and tax rebate pledgeability $\psi=\tilde{E}/E(X^*,I_1^*)$.*

This implies that taxes and SRI premiums are imperfect substitutes in incentivizing borrowers to abate. While the direct incentive effect of τ and ω on borrowers' abatement choice is the same, SRIs' overall impact on financial constraints depends on their benchmark $\tilde{E}$, while the effect of the tax depends on the size of pledgeable tax rebates ψ.

An implication is that, if $\tilde{E}$ is sufficiently low, the presence of SRIs may worsen the trade-offs faced by a regulator setting emissions taxes, due to the tightening of borrowers' financial constraints. This outcome occurs if SRIs charge higher financing costs to brown firms (for instance, when $\tilde{E}=0$), consistent with popular real-world ESG investing strategies that focus on divesting from brown firms. This result shows that divestment strategies may backfire if they increase brown firms' cost of capital, consistent with evidence in Hartzmark and Shue (2023)

SRIs can be more beneficial if instead investor preferences lead to favorable financing terms and looser financial constraints, i.e. if $\tilde{E}$ is high. This outcome occurs if investors have a more impact-oriented objective and derive positive utility from downward deviations relative to a benchmark level, or, more broadly, if SRIs derive utility from reforming firms (as in, e.g. Allen et al., 2023; Green and Roth, 2025; Gupta et al., 2025). While analyzing different SRI preferences in greater detail is beyond the scope of this paper (for a comprehensive analysis of different types of investors' pro-social preferences, see Dangl et al., 2024), our result highlights that understanding the nature of SRI preferences and strategies is critical for assessing their real-world impact and the nature of their interactions with environmental policies.

5.2.2. Hedging and climate-linked bonds

In the baseline model, there is no risk. Yet, in reality there is substantial uncertainty about the future impacts of climate change, as evident in the wide range of estimates of the social cost of carbon (Golosov et al., 2014; Nordhaus, 2019). In this extension we introduce climate risk by assuming that at $t=1$ all agents learn whether the economy is in a good state ($s=G$) or in a bad state ($s=B$). In the bad state γ_s takes a high value $\gamma_B>\gamma_G$. The probability of the bad state is given by q_B and that of the good state is equal to $q_G=1-q_B$.

In the Internet Appendix (Section IA.5), we characterize the problem of borrowers and show how optimal emissions pricing is now state-contingent and may be highly constrained in the bad state of the world. This is because borrowers' choice of leverage now depends on the *expected* cost of binding financial constraints, which implies that from the ex-post perspective leverage is excessive in the bad state, which can result in a high shadow cost of a binding financial constraints, $\lambda_B>\lambda_G$.

We then consider fairly-priced hedging contracts that pay h_B in the bad state and h_G in the good state. Such contracts can be implemented through carbon price derivatives, or through state-contingent financing such as "climate linkers" that write off the principal by h_B when carbon taxes (or the social cost of emissions) are high, in return for an interest payment h_G when taxes are low. Fair pricing requires that

$$(1-q_B)h_G+q_Bh_B=0. \tag{22}$$

The borrower's problem now includes additional choices h_G and h_B. In the Internet Appendix we show that borrowers optimally shift resources from the good to the bad state, such that $\lambda_G=\lambda_B$. If this allows borrowers to ensure that financial constraints are slack in both states ($\lambda_G=\lambda_B=0$), then a Pigouvian emissions tax $\tau_s=\gamma_s, \forall s\in\{B,G\}$ can implement the first-best allocation (see Proposition 2). This implies that, by allowing firms to hedge climate-related transition risk, the financial sector can enable efficient emissions taxation in equilibrium.

This highlights that hedging of climate-related risks may be an important role the financial sector can play in supporting the transition to a low-carbon economy, distinct from socially responsible investing. We also contribute to the nascent debate on climate-linked securities. Our analysis shows that supporting such markets can allow more efficient environmental policy in equilibrium, thus pointing to benefits that go beyond the direct risk-sharing and informational gains discussed so far (see Chikhani and Renne, 2022).

Some degree of hedging climate risks could also be achieved using external equity or long-term debt. However, we show in the Internet Appendix (Sections IA.5.3 and IA.5.4) that the risk-sharing benefits are more limited compared to carbon price hedging.

5.3. Multiple jurisdictions

In the baseline model, environmental policy is set by a global regulator in a single jurisdiction or, equivalently, corresponds to a setting with multiple jurisdictions that have one perfectly coordinated policy. However, in reality international coordination is an important friction undermining the full internalization of global emissions externalities. In this extension, we assume that there are two jurisdictions, j and $-j$. A fraction α of borrowers and investors are based in jurisdiction j, and the remaining agents are in $-j$. Total emissions in the respective jurisdictions are denoted by $E^j=\alpha E(X_j,I_{1,j})$ and $E^{-j}=(1-\alpha)E(X_{-j},I_{1,-j})$, respectively, and aggregate global emissions are $E^a=E^j+E^{-j}$. In each jurisdiction, a regulator sets emissions taxes so as to maximize the welfare of all agents in the jurisdiction, ignoring externalities on agents in the other jurisdiction. For tractability, we assume in this extension that $-\partial R(I_1,E^a)/\partial E^a=\gamma^p$ is a constant, and we focus on the case in which borrowers in $-j$ are not financially constrained ($\lambda_{-j}=0$; the case $\lambda_{-j}>0$ is discussed in the Internet Appendix).

In the Internet Appendix (Section IA.7), we formally set up and solve the agents' and regulators' problems, and derive the following result comparing the multi-jurisdiction emissions tax to the emissions tax set in the baseline model with a single jurisdiction.

Proposition 7. *If $\gamma^p = 0$ or $\lambda_j = 0$, then the optimal emissions tax in jurisdiction j is independent of E^{-j} and equivalent to the one in the baseline model with social costs of emissions $\tilde{\gamma}^u = \alpha\gamma^u$ instead of γ^u and $\tilde{\gamma}^p = \alpha\gamma^p$ instead of γ^p.*

If $\gamma^p > 0$ and $\lambda_j > 0$, then the same mapping holds if additionally borrowers' pledgeable income in the baseline model is reduced by $\theta\gamma^p E^{-j}$.

Proof. See Internet Appendix Section IA.7. □

As regulators do not internalize effects on agents in other jurisdictions, they set laxer regulation than under full coordination. With slack financial constraints, this results in an emissions tax $\tau_j^* = 2\tilde{\gamma}^u + \tilde{\gamma}^p = \alpha\gamma$, which is independent of regulation in jurisdiction $-j$ and scales with the size of jurisdiction j because regulators in a larger jurisdiction internalize more of the externality. This effect resembles the standard coordination problem in global externalities, highlighting the benefits of international coordination (for example, see Nordhaus, 2019).

The second part of the proposition highlights an additional global spillover due to the climate-induced collateral externality present in our model. With binding financial constraints and $\gamma^p > 0$, emissions in one jurisdiction have a negative impact on the pledgeable income of borrowers in other jurisdictions due to the physical climate impacts on pledgeable resources. As we show in the Internet Appendix, this implies that higher emissions in jurisdiction $-j$ lead to more liquidations in jurisdiction j. Thus, the collateral externality leads to a financial spillover between jurisdictions, captured by the additional loss in pledgeable income of $\theta\gamma^p E^{-j}$. This loss is driven by emissions in jurisdiction $-j$, which are outside the direct control of the regulator in jurisdiction j. Consequently, from the perspective of the regulator in j, this loss constitutes an exogenous tightening in financial constraints, leading to more liquidations and worsening the trade-offs faced by a regulator setting emissions taxes. Thus, climate-induced collateral externalities give rise to financial spillovers across jurisdictions, exacerbating the costs of insufficient global policy coordination on climate change.

An implication of Proposition 7 is that, keeping the size of jurisdiction j fixed at α, financial spillovers from multiple small jurisdictions may be larger than from one single other jurisdiction. To see this, suppose that all $1-\alpha$ agents outside of jurisdiction j are evenly distributed across $J \geq 1$ other jurisdictions. As the number of jurisdictions J increases, the internalized social cost of emissions $(1-\alpha)\gamma/J$ targeted by each regulator outside of j declines. Consequently, total emissions by all jurisdictions other than j may increase in the number of jurisdictions, worsening the trade-offs faced by the regulator in j. Hence, in the presence of climate-induced collateral externalities, the optimal policy of the regulator in a large economy depends on whether the remaining jurisdictions are fragmented or coordinate.

6. Conclusion

This paper provides an analytical framework to shed light on how emissions pricing interacts with financial constraints. We uncover a climate-related collateral externality that affects how emissions taxes interact with financial constraints. Higher emissions taxes tighten financial constraints if borrowers have carbon-emitting assets, but emissions taxes can ease financial constraints if they have a positive effect on the collateral value of assets exposed to physical climate impacts. This highlights that the interaction between financial constraints and emissions pricing is nuanced, and that optimal emissions pricing needs to account for climate-induced collateral externalities.

The fact that financial constraints distort Pigouvian emissions pricing does not necessarily imply that regulators can do better by using other policy tools. Emissions pricing alone can result in a constrained-efficient allocation if implemented through emissions taxes when tax rebates are fully pledgeable, or if implemented through a cap-and-trade system with ex-ante freely allocated pollution permits. Only if such policies are not available, it may be beneficial to consider climate-related goals in other policy tools, for example, by complementing carbon taxes with leverage regulation. Fostering financial markets that allow firms to hedge regulatory risk, such as carbon-price derivatives or climate-linked bonds, can improve equilibrium climate policies by enabling firms to shoulder higher carbon prices.

CRediT authorship contribution statement

Robin Döttling: Writing – review & editing, Writing – original draft, Project administration, Methodology, Investigation, Formal analysis. **Magdalena Rola-Janicka:** Writing – review & editing, Writing – original draft, Project administration, Methodology, Investigation, Formal analysis.

Declaration of competing interest

I have nothing to disclose.

Appendix A. First best and competitive equilibrium

To simplify the notation in parts of the Appendix we use the following definition

$$N(X, I_1, \tau) = -\tau E(X, I_1) - C(X, I_1). \tag{A.1}$$

Moreover, to make the expressions more legible we sometimes use the following shorthand notation: $F(X, I_1) = F$, $F'_X = \partial F(X, I_1)/\partial X$, $F'_I = \partial F(X, I_1)/\partial I_1$ for $F = E$ and $F = C$, $N''_{XI} = \partial^2 N(X, I_1, \tau)/(\partial X \partial I_1)$, $\partial R(I_1, E^a)/\partial I_1 = \rho$.

A.1. First best (Proof of Proposition 1)

This appendix proves Proposition 1. To distinguish investors and borrowers, we use super-scripts i and b, respectively. The first-best allocation maximizes social welfare defined as $W = U^i + U^b$, subject to aggregate resource constraints at $t = 1, 2$

$$c_1^b + c_1^i + C(X, I_1) = A^i + \mu(I_0 - I_1)$$
$$c_2^b + c_2^i = R(I_1, E^a)$$

Using the resource constraints to eliminate $c_1^i + c_1^b$ and $c_2^i + c_2^b$, the problem can be written as the following Lagrangian:

$$\max_{I_1, X} \mathcal{L} = A^i + \mu(I_0 - I_1) - C(X, I_1) + R(I_1, E^a) - \gamma^u E(X, I_1) + \underline{\kappa}_I I_1 + \overline{\kappa}_I (I_0 - I_1),$$

with $\underline{\kappa}_I$ and $\overline{\kappa}_I$ the Lagrange multipliers on the constraint that $I_1 > 0$ and $I_1 \leq I_0$, respectively. The FOC's with respect to X, and I_1 are, respectively:

$$\gamma \frac{\partial E(X, I_1)}{\partial X_1} + \frac{\partial C(X, I_1)}{\partial X_1} = 0,$$
$$r(\gamma, X, I_1) + \underline{\kappa}_I - \overline{\kappa}_I = 0.$$

The first condition is the one stated in Proposition 1. The second condition can be used to show that in the first best it must be that $\overline{\kappa}_I > 0$, which implies that $I_1 = I_0$ as stated in Proposition 1. To see this, recall that by Assumption 2.1 liquidations are privately inefficient for any $\tau \leq \bar{\tau}$, with $\bar{\tau} > \gamma$. This implies that

$$r(\gamma, X, I_1) = \frac{\partial R(I_1, E^a)}{\partial I_1} - \mu - \frac{\partial C(X, I_1)}{\partial I_1} - \gamma \frac{\partial E(X, I_1)}{\partial I_1} > 0$$

Hence, the planner's first order condition w.r.t. I_1 can only be satisfied if $\bar{\kappa}_I > 0$, which implies that $I_1 = I_0$ in the first-best allocation.

A.2. Borrower's Lagrangian

This appendix formally states the Lagrangian for the borrower's problem in Section 3.1, from which the first order conditions in Section 3.2 are derived.

The financial constraint (5) implies $c_2 > 0$, so that the non-negativity constraint on c_2 never binds. Thus, after eliminating c_1, and c_2 using Eqs. (3) and (4), the problem of borrowers can be stated as:

$$\max_{X,I_1,d_1} \mathcal{L} = \mu(I_0 - I_1) - C(X,I_1) + R(I_1,E^a) - \tau E(X,I_1) + T + \lambda\left[\theta R(I_1,E^a) - \tau E(X,I_1) + \psi T - d_1\right] + \underline{\kappa}_I I_1 + \overline{\kappa}_I[I_0 - I_1] + \kappa_{c_1}\left[d_1 + \mu(I_0 - I_1) - C(X,I_1)\right]. \tag{A.2}$$

The first order condition w.r.t. d_1 implies that $\lambda = \kappa_{c_1}$. The remaining FOC's of the problem are given in Section 3.2.

A.3. Proof of Lemma 1

With a slack financial constraint, Eq. (9) evaluated at $\lambda = 0$ is $r(\tau, X, I_1) - \overline{\kappa}_I + \underline{\kappa}_I = 0$. By Assumption 2.1 we have that $r(\tau, X, I_1) > 0$, which implies that the solution requires $\overline{\kappa}_I > 0$ (i.e., $I_0 = I_1^*$).

With a binding financial constraint, the complementary slackness condition (10) can be reformulated as

$$\lambda S(\tau, X, I_1) = 0,$$

where $S(\tau, X, I_1) \equiv \theta R(I_1, E^a) - \tau E(X, I_1) + \psi T + \mu(I_0 - I_1) - C(X, I_1)$ collects the terms in square brackets in Eq. (10). By Assumption 2.2, liquidating investments eases financial constraints, so $\partial S/\partial I_1 < 0$. If $S(\tau, X, I_1 = I_0) < 0$, financial constraints bind, $\lambda > 0$. In this case, the complementary slackness condition requires that borrowers choose I_1^* s.t. $S(\tau, X, I_1^*) = 0$. Thus, if $\lambda > 0$ it must be that $I_1^* < I_0$ and $\overline{\kappa}_I = 0$.

Appendix B. Optimal policy

B.1. Effect of emissions tax on abatement

Totally differentiating Eq. (8) with respect to τ allows us to find $\partial X^*/\partial\tau$:

$$\frac{\partial X^*}{\partial \tau} = \frac{\frac{\partial E(X,I_1)}{\partial X} - \frac{\partial^2 N(X,I_1,\tau)}{\partial X \partial I_1}\frac{\partial I_1^*}{\partial \tau}}{-\frac{\partial^2 C(X,I_1)}{\partial X^2}} \tag{B.1}$$

where we use the definition of $N(X, I_1, \tau)$ from Eq. (A.1) and that by Assumption 1.3 $\partial^2 E(X, I_1)/\partial X^2 = 0$.

If the financial constraint is slack, $\lambda^*(\tau) = 0$, then $I_1^* = I_0$, so that $\partial I_1^*/\partial\tau = 0$. Together with the fact that $\partial^2 C(X, I_1)/\partial X^2 > 0$ by Assumption 1.3, this implies $\partial X^*/\partial\tau > 0$ in this case (without further parameter conditions).

If the financial constraint binds, $\lambda^*(\tau) > 0$, the sign of $\partial X^*/\partial\tau$ is less straightforward. In this case, $\partial I_1^*/\partial\tau$ follows from totally differentiating Eq. (10) with respect to τ:

$$\frac{\partial I_1^*}{\partial \tau} = \frac{(1-\psi)E(X^*, I_1^*) - (\psi\tau - \theta\gamma^p)\frac{\partial E(X,I_1)}{\partial X}\frac{\partial X^*}{\partial \tau}}{\tilde{r}(\tau(1-\psi) + \theta\gamma^p, X^*, I_1^*)} \tag{B.2}$$

where $\gamma^p = -\partial R(I_1, E^a)/\partial E^a$.

In the paper, we focus on the case in which emissions taxes incentivize abatement, i.e. $\partial X^*/\partial\tau > 0$. This is the case if the optimal abatement is not too sensitive to the level of investment, i.e. the second order effects of increasing τ on abatement operating through the effect of τ on the optimal investment scale are not too high relative to the direct incentive effect. Below we first derive the restrictions on the functional forms that ensure that this is the case and then characterize equilibrium outcomes if these conditions are not satisfied and instead $\partial X^*/\partial\tau < 0$.

Conditions for $\partial X^*/\partial\tau > 0$. We combine (B.1) and (B.2) to isolate the terms $\partial I_1^*/\partial\tau$ and $\partial X^*/\partial\tau$:

$$\frac{\partial I_1^*}{\partial \tau} = \frac{(1-\psi)EC''_{X^2} + (\psi\tau - \theta\gamma^p)(E'_X)^2}{\tilde{r}(\tau(1-\psi) + \theta\gamma^p, X^*, I_1^*)C''_{X^2} + (\psi\tau - \theta\gamma^p)E'_X N''_{XI}}, \tag{B.3}$$

$$\frac{\partial X^*}{\partial \tau} = \frac{(1-\psi)EN''_{XI} - \tilde{r}(\tau(1-\psi) + \theta\gamma^p, X^*, I_1^*)E'_X}{\tilde{r}(\tau(1-\psi) + \theta\gamma^p, X^*, I_1^*)C''_{X^2} + (\psi\tau - \theta\gamma^p)E'_X N''_{XI}}, \tag{B.4}$$

where we simplified the expressions using the shorthand notation for E'_X, C'_X, N''_{XI}, etc., introduced at the beginning of the Appendix.

To focus on model parameters such that $\partial X^*/\partial\tau > 0$, the numerator and denominator of (B.4) should have the same sign. While it is not possible to provide explicit parameter conditions under which this is always the case, it can easily be seen that the denominator of (B.4) is negative if $\psi = 0$ and $\gamma^p = 0$, because $C''_{X^2} > 0$ and $\tilde{r}(\tau, X^*, I_1^*) < 0$ by Assumptions 1.3 and 2.2 respectively. Therefore, we focus on parameter ranges such that both the denominator and the numerator of (B.4) are negative. This implicitly defines the parameters required for $\partial X^*/\partial\tau > 0$.

To further characterize the conditions, note the denominator is negative if and only if $\tilde{r}(\tau(1-\psi)+\theta\gamma^p, X^*, I_1^*)C''_{X^2} < -(\psi\tau - \theta\gamma^p)N''_{XI}E'_X$. For the numerator, we can use Definition 1 to expand $\tilde{r}(\tau(1-\psi) + \theta\gamma^p, X^*, I_1^*) = \tilde{r}(\theta\gamma^p, X^*, I_1^*) - (1-\psi)\tau E'_I$. Using this in the numerator of (B.4), we can see that it is negative whenever $\tilde{r}(\theta\gamma^p, X^*, I_1^*)E'_X > (1-\psi)(\tau E'_I E'_X + EN''_{XI})$. This condition is always satisfied if $\psi = 1$, using Assumption 2.2, and since the RHS of this inequality is monotone in ψ, the numerator of (B.4) is negative for any ψ if the inequality also holds for $\psi = 0$, i.e. if $\tilde{r}(\theta\gamma^p, X^*, I_1^*)E'_X > \tau E'_I E'_X + EN''_{XI}$. Thus, to ensure that $\partial X^*/\partial\tau > 0$ we make the following assumption.

Assumption 3. Model parameters are such that $\forall X^*(\tau), I_1^*(\tau), \tau < \bar{\tau}$:

$$\tilde{r}(\theta\gamma^p, X^*, I_1^*)\frac{\partial E(X,I_1)}{\partial X} > \tau\frac{\partial E(X,I_1)}{\partial X}\frac{\partial E(X,I_1)}{\partial I_1} + E(X^*, I_1^*)\frac{\partial^2 N(X,I_1,\tau)}{\partial X \partial I_1}$$

$$\tilde{r}(\tau(1-\psi) + \theta\gamma^p, X, I_1)\frac{\partial^2 C(X,I_1)}{\partial X^2} < -(\psi\tau - \theta\gamma^p)\frac{\partial^2 N(X,I_1,\tau)}{\partial X \partial I_1}\frac{\partial E(X,I_1)}{\partial X}$$

Equilibrium if $\partial X^*/\partial\tau < 0$. In the main text we focus on the interesting case $\partial X^*/\partial\tau > 0$, which holds under Assumption 3. For completeness, here we discuss the equilibrium outcomes if instead $\partial X^*/\partial\tau < 0$. Notice that when the tax decreases abatement, higher τ is associated with losses through the collateral externality channel and gains due to higher tax rebates. Below we characterize how these interactions affect the sensitivity of investment scale to the tax when $\partial X^*/\partial\tau < 0$.

- If $(1-\psi)E(X^*, I_1^*) + \theta\gamma^p\partial E^a/\partial X \times \partial X^*/\partial\tau < \psi\tau\partial E^a/\partial X \times \partial X^*/\partial\tau$ then $\partial I_1^*/\partial\tau > 0$.
- If $(1-\psi)E(X^*, I_1^*) + \theta\gamma^p\partial E^a/\partial X \times \partial X^*/\partial\tau > \psi\tau\partial E^a/\partial X \times \partial X^*\partial\tau$ then $\partial I_1^*/\partial\tau < 0$.

Using this in the regulator's FOC (11) implies that:

- If $\partial I_1^*/\partial\tau < 0$ then there are two solutions to (11): $\tau = 0$ and potentially $\tau > \gamma$. Note that if the left-hand-side of (11) crosses 0 at some $\tau > \gamma$ it is crossing it from below, so the solution minimizes welfare. Given the constraint that $\tau \geq 0$ welfare is maximized at $\tau = 0$
- If $\partial I_1^*/\partial\tau > 0$ then the equilibrium must feature: $\tau < \gamma$. In this case the optimal emissions tax may be above 0 as a higher tax relaxes financial constraints.

B.2. Proof of Lemma 2

The derivative $\partial I_1^*/\partial\tau$ is defined in Eq. (B.3). The denominator of (B.3) is negative under Assumption 3 stated in Appendix B.1. Lemma 2 follows from observing that the numerator of Eq. (B.3) is negative if

$-\partial R(I_1, E^a)/\partial E^a = \gamma^p > \hat{\gamma}^p(\tau)$ and positive if $-\partial R(I_1, E^a)/\partial E^a = \gamma^p < \hat{\gamma}^p(\tau)$, where

$$\hat{\gamma}^p(\tau) \equiv \frac{\psi}{\theta}\tau + \frac{(1-\psi)E(X^*, I_1^*)\frac{\partial^2 C(X,I_1)}{\partial X^2}}{\theta\left(\frac{\partial E(X,I_1)}{\partial X}\right)^2}. \tag{B.5}$$

B.3. Proof of Proposition 3

The regulator's problem is to set the emissions tax τ so as to maximize welfare at $t = 1$ $W_1 = c_1^i + c_2^i + c_1^b + c_2^b - 2\gamma^u E^a$ (where we use super-scripts i and b to distinguish between investors and borrowers), subject to the non-negativity constraint on τ. We eliminate c_1^b, and c_2^b using Eqs. (3) and (4), and substitute $c_1^i = A^i - d_1$, and $c_2^i = d_1$, to write the regulator's problem as the following Lagrangian:

$$\max_{\tau} \quad A^i + R(I_1^*, E^a) + \mu(I_0 - I_1^*) - 2\gamma^u E^a - C(X^*, I_1^*) + \kappa_\tau \tau. \tag{B.6}$$

The first order condition with respect to τ is given by:

$$\left(\gamma\frac{\partial E(X,I_1)}{\partial X} + \frac{\partial C(X,I_1)}{\partial X}\right)\frac{\partial X^*}{\partial \tau} = \left(\frac{\partial R(I_1,E^a)}{\partial I_1} - \mu - \gamma\frac{\partial E(X,I_1)}{\partial I_1} - \frac{\partial C(X,I_1)}{\partial I_1}\right)\frac{\partial I_1^*}{\partial \tau} + \kappa_\tau. \tag{B.7}$$

Using (8) and the definition of $r(\tau, X, I_1)$ the FOC above simplifies to (11). In Eq. (11), the term $\partial E(X, I_1)/\partial X \times \partial X^*/\partial\tau < 0$ under Assumption 1.1 and given we focus on $\partial X^*/\partial \tau > 0$ (which holds under Assumption 3 stated in Appendix B.1), while $r(\gamma, X, I_1) > 0$ by Assumption 2. Consequently, for Eq. (11) to hold the optimal tax must be:

- lower than the direct social cost of carbon $\tau < \gamma$ if $\partial I_1^*/\partial\tau < 0$
- equal to the direct social cost of carbon $\tau = \gamma$ if $\partial I_1^*/\partial\tau = 0$
- higher than the direct social cost of carbon $\tau > \gamma$ if $\partial I_1^*/\partial\tau > 0$

The result in Proposition 3 in terms of the threshold $\hat{\gamma}^p(\tau)$ follows from using Lemma 2 to determine the sign of $\partial I_1^*/\partial\tau$.

Appendix C. Constrained efficiency

C.1. Proof of τ^* vs. τ^{GP} result in Proposition 4

Using Eq. (12) in Eq. (11) and the definition of λ^* in the interior solution for I_1, the regulator's FOC can be rewritten as:

$$r(\gamma, X^*, I_1^*)(1-\psi)E(X^*, I_1^*) + \kappa_\tau \tilde{r}(\tau(1-\psi) + \theta\gamma^p, X^*, I_1^*) = \left[\gamma - \tau + \lambda^*(\theta\gamma^p - \psi\tau)\right]\frac{\partial E(X,I_1)}{\partial X}\frac{\partial X^*}{\partial\tau}\tilde{r}(\tau, X^*, I_1^*) \tag{C.1}$$

The RHS of Eq. (C.1) is zero if $\tau = \tau^{GP} \equiv (\gamma + \lambda^*\theta\gamma^p)/(1+\psi\lambda^*)$. The RHS is positive whenever $\tau < \tau^{GP}$, since $\partial E(X, I_1)/\partial X < 0$, $\tilde{r}(\tau, X^*, I_I^*) < 0$ and $\partial X^*/\partial\tau > 0$ under Assumptions 1.1, 2.2 and 3 respectively.

In the interior solution, $\kappa_\tau = 0$ **and** $\tau > 0$. If $\psi = 1$, then the LHS of (C.1) is equal to zero, so the optimal emissions tax must be $\tau = \tau^{GP}$. If $\psi < 1$, then the LHS is positive, so the optimal emissions tax must satisfy $\tau < \tau^{GP}$.

In the corner solution, $\kappa_\tau > 0$ **and** $\tau = 0$. We first show that $\tau = 0$ while $\gamma > 0$ can only be an equilibrium if $\psi < 1$. We do it in two steps: (i) show that if $\psi = 1$ and $\gamma > 0$, then $\tau^* = 0$ cannot be an equilibrium and (ii) show that if $\psi < 1$ and $\gamma > 0$, then $\tau^* = 0$ is a feasible equilibrium. Then we show that when $\psi < 1$ and $\gamma > 0$, the equilibrium tax satisfies $\tau^* < \tau^{GP}$ as stated in Proposition 4.
(i) Notice that when $\tau = 0$ and $\gamma > 0$, then the RHS of Eq. (C.1) is strictly positive. If $\psi = 1$, the LHS of Eq. (C.1) is weakly negative, since $\tilde{r}(\tau(1-\psi) + \theta\gamma^p, X^*, I_I^*) < 0$ and $\kappa_\tau \geq 0$. Thus, if $\psi = 1$ and $\gamma > 0$, $\tau^* = 0$ cannot be an equilibrium.
(ii) If $\psi < 1$, the LHS of Eq. (C.1) can take any sign, depending on the relative size of κ_τ. Since the RHS of Eq. (C.1) is positive whenever $\tau^* = 0 < \gamma$, such equilibrium is feasible.
(iii) Since $\tau^{GP}(\gamma) > 0 \; \forall \; \gamma > 0$, it follows that $\tau^* = 0 < \gamma$ when $\psi < 1$ is consistent with $\tau^* < \tau^{GP}$.

C.2. Proof of constrained efficiency result in Proposition 4

We define the constrained-efficient allocation in which a social planner can choose X and I_1 directly without any policy instruments, but subject to the same constraints as private agents. Using i and b super-scripts to denote investors and borrowers, we eliminate c_1^b, and c_2^b using Eqs. (3) and (4), and use $c_1^i = A^i - d_1$, and $c_2^i = d_1$, to write the planner's problem as the following Lagrangian:

$$\begin{aligned}\max_{X,I_1} \; \mathcal{L} = & A^i + R(I_1, E^a) + \mu(I_0 - I_1) - 2\gamma^u E(X, I_1) - C(X, I_1) \\ & + \lambda^{SP}\left[\theta R(I_1, E^a) + \mu(I_0 - I_1) - C(X, I_1)\right] \\ & + \underline{\kappa}_I^{SP} I_1 + \overline{\kappa}_I^{SP}(I_0 - I_1).\end{aligned} \tag{C.2}$$

The constrained-efficient levels of $I_1^{SP}, X^{SP}, \lambda^{SP}$ are pinned down by the FOCs with respect to X and I_1 and the complementary slackness condition:

$$-(\gamma + \lambda^{SP}\theta\gamma^p)\frac{\partial E(X,I_1)}{\partial X} - (1+\lambda^{SP})\frac{\partial C(X,I_1)}{\partial X} = 0, \tag{C.3}$$

$$r(\gamma, X, I_1) + \lambda^{SP}\tilde{r}(\gamma^p, X, I_1) - \overline{\kappa}_I^{SP} + \underline{\kappa}_I^{SP} = 0, \tag{C.4}$$

$$\lambda^{SP}[\theta R(I_1, E^a) + \mu(I_0 - I_1) - C(X, I_1)] = 0. \tag{C.5}$$

To check whether the equilibrium is constrained efficient, we compare the planner's FOCs to the borrowers FOCs (8) and (9) and the complementary slackness condition (10). The equilibrium is constrained efficient if and only if $X^*(\tau^*) = X^{SP}$, $I_1^*(\tau^*) = I_1^{SP}$ and $\lambda^*(\tau^*) = \lambda^{SP}$. This is the case if (8) is equivalent to (C.3), (9) is equivalent to (C.4), and (10) is equivalent to (C.5). To check whether this is the case, we postulate that $X^*(\tau^*) = X^{SP}$, $I_1^*(\tau^*) = I_1^{SP}$ and $\lambda^*(\tau^*) = \lambda^{SP}$, and verify whether each of the borrower-planner FOC pairs are equivalent given τ^* defined in Eq. (11).

Case $\psi < 1$.

- If (C.5) is satisfied at $X^{SP} = X^*, I_1^{SP} = I_1^*$, then (10) is satisfied if and only if $\tau^* E(X^*, I_1^*) - \psi T^* = 0$. This is the case only if $\tau^* = 0$ and $T^* = 0$.
- If $I_1^{SP} = I_1^*$, then (C.3) is equivalent to (8) if and only if $\tau^* = (\gamma + \lambda^{SP}\theta\gamma^p)/(1 + \lambda^{SP}) \equiv \tau^{SP}$.

Thus, for $X^{SP} = X^*$, $I_1^{SP} = I_1^*$ and (C.5) to be equivalent to (10), it must be that $\tau^{SP} = 0$. However, this is the case only if $\gamma = 0$, contradicting that $\gamma > 0$. Hence, if $\psi < 1$ the competitive equilibrium is not constrained efficient.

Case $\psi = 1$. We proceed in four steps:

1. (10) & (C.5): When $\psi = 1$, then $-\tau^* E(X^*, I_1^*) + \psi T^* = 0$. This implies (10) is equivalent to (C.5).
2. (8) & (C.3): The two conditions are equivalent if $\tau^* = (\gamma + \lambda^{SP}\theta\gamma^p)/(1+\lambda^{SP}) \equiv \tau^{SP}$. We have shown above that the optimal emissions tax is given by $\tau^* = \tau^{GP} = (\gamma + \lambda^*\theta\gamma^p)/(1 + \lambda^*)$ when $\psi = 1$. This implies that $\tau^* = \tau^{SP}$ whenever $\lambda^*(\tau^*) = \lambda^{SP}$. We show that this holds below.
3. (9) & (C.4): the two conditions are equivalent if and only if $\lambda^{SP} = \lambda^*(\tau^*)$. Verifying that $\lambda^{SP} = \lambda^*(\tau^*)$ at $\tau^* = \tau^{SP}$ also establishes that (8) is equivalent to (C.3) (see step 2).

To verify that $\lambda^*(\tau^{SP}) = \lambda^{SP}$, we first find τ^{SP} and then plug it into borrower's FOC (9) to find $\lambda^*(\tau^{SP})$. Eq. (C.4) implies that $\lambda^{SP} =$

$r(\gamma, X^{SP}, I_1^{SP})/\tilde{r}(\theta\gamma^p, X^{SP}, I_1^{SP})$. Using this in the expression for τ^{SP}:

$$\begin{aligned}\tau^{SP} &= \frac{\gamma\tilde{r}(\theta\gamma^p, X^{SP}, I_1^{SP}) - \theta\gamma^p r(\gamma, X^{SP}, I_1^{SP})}{\tilde{r}(\theta\gamma^p, X^{SP}, I_1^{SP}) - r(\gamma, X^{SP}, I_1^{SP})} \\ &= \frac{\rho\theta(\gamma - \gamma^p) - w(\gamma - \theta\gamma^p)}{\rho(\theta - 1) - (\theta\gamma^p - \gamma)E_I'}\end{aligned} \tag{C.6}$$

where $w = \partial C(X, I_1)/\partial I_1 + \mu$ and we use short-hand notation introduced at the beginning of the Appendix: $E_I' = \partial E(X, I_1)/\partial I_1$ and $\rho = \partial R(I_1, E^a)/\partial I_1$.

Using the expression for τ^{SP} in (18) to find $\lambda^*(\tau^{SP})$:

$$\begin{aligned}\lambda^*(\tau^{SP}) &= -\frac{r(\tau^{SP}, X^*, I_1^*)}{\tilde{r}(\tau^{SP}, X^*, I_1^*)} = -\frac{\rho - w - \tau^{SP}E_I'}{\theta\rho - w - \tau^{SP}E_I'} \\ &= -\frac{\rho^2(\theta - 1) - w\rho(\theta - 1) - \rho\gamma(\theta - 1)E_I'}{\theta\rho^2(\theta - 1) - w\rho(\theta - 1) - \rho\theta\gamma^p(\theta - 1)E_I'} \\ &= -\frac{\rho - w - \gamma E_I'}{\theta\rho - w - \theta\gamma^p E_I'} = -\frac{r(\gamma, X^{SP}, I_1^{SP})}{\tilde{r}(\theta\gamma^p, X^{SP}, I_1^{SP})} = \lambda^{SP}\end{aligned}$$

where we use that $X^* = X^{SP}$, $I_1^* = I_1^{SP}$ and the last step follows from the definition of λ^{SP} for the interior solution of I_1 in Eq. (C.4). This completes the proof that $\lambda^*(\tau^{SP}) = \lambda^{SP}$.

Appendix D. Cap-and-trade

This appendix derives the borrower's problem under the cap-and-trade system laid out in Section 4.3, and derives the optimal permit price. We assume here that the proceeds from the sale of permits are distributed to investors (Internet Appendix IA.2.3 shows that the insights on implementing the constrained efficient allocation are robust if sale proceeds are distributed to borrowers instead). The budget constraints of the borrower under the pollution trading scheme are:

$$c_1^b = \mu(I_0 - I_1) + d_1 - C(X, I_1) \geq 0, \tag{D.1}$$

$$c_2^b = R(I_1, E^a) - (1 - \phi)Qp + p(Q - E(X, I_1)) - d_1 \geq 0, \tag{D.2}$$

$$d_1 \leq \theta R(I_1, E^a). \tag{D.3}$$

The borrower's problem is analogous to the one with emissions taxes, but with the pollution permit price p taking the place of the tax τ, as shown in the budget constraints above. The FOCs and the complementary slackness condition are:

$$(1 + \lambda)\left(p\frac{\partial E(X, I_1)}{\partial X} + \frac{\partial C(X, I_1)}{\partial X}\right) = 0, \tag{D.4}$$

$$\rho(1 + \lambda\theta) - (1 + \lambda)\left[\mu + \frac{\partial C(X, I_1)}{\partial I_1} + p\frac{\partial E(X, I_1)}{\partial I_1}\right] - \overline{\kappa}_I + \underline{\kappa}_I = 0, \tag{D.5}$$

$$\lambda[\theta R(I_1, E^a) + \mu(I_0 - I_1) - C(X, I_1) + p(\phi Q - E(X, I_1))] = 0. \tag{D.6}$$

Regulator problem. The regulator sets the amount of emissions Q. Condition (D.4), together with the market clearing for permits, $Q = E^a$, jointly determine a mapping from p to E^a. Thus, the regulator can implement a desired market price of permits by altering the total quantity of permits. Consequently, we can solve the regulator's problem as maximizing social welfare at $t = 1$ by choosing p, analogous to the regulator problem with emission taxes in Eq. (B.6). The first order condition of the regulator is:

$$r(\gamma, X^*, I_1^*)\frac{\partial I_1^*}{\partial p} - (\gamma - p)\frac{\partial E(X, I_1)}{\partial X}\frac{\partial X^*}{\partial p} + \kappa_p = 0 \tag{D.7}$$

To find $\partial X^*/\partial p$, we take a total derivative of (D.4) with respect to p. This yields:

$$\frac{\partial X^*}{\partial p} = \frac{\frac{\partial E(X,I_1)}{\partial X} - \frac{\partial^2 N(X,I_1,p)}{\partial X\partial I_1}\frac{\partial I_1}{\partial p}}{\frac{\partial^2 C(X,I_1)}{\partial X^2}} \tag{D.8}$$

To find $\partial I_1^*/\partial p$ take a total derivative of (D.6) with respect to p, keeping in mind that $\phi Q = \phi E^a$.

$$\frac{\partial I_1^*}{\partial p} = \frac{(1 - \phi)E(X^*, I_1^*) - (\phi p - \theta\gamma^p)\frac{\partial E(X,I_1)}{\partial X}\frac{\partial X^*}{\partial p}}{\tilde{r}(p(1 - \phi) - \theta\gamma^p, X^*, I_1^*)} \tag{D.9}$$

In the baseline model we can define $\partial X^*/\partial\tau = g_X(\tau, \psi)$ and $\partial I_1^*/\partial\tau = g_I(\tau, \psi)$. Comparing (B.1) with (D.8) and (B.2) with (D.9), it is straightforward that $\partial X^*/\partial p = g_X(p, \phi)$ and $\partial I_1^*/\partial p = g_I(p, \phi)$. Thus, the first order condition of the regulator's problem in the baseline model (11) is equivalent to the first order condition of the problem of choosing Q to implement p taking as given ϕ, given by (D.7). The two problems are exactly the same if $\psi = \phi$. This proves the statement in Proposition 5.

Effect of free permits on aggregate emissions (Lemma 3). We now turn to understanding the effect of free allocation of permits on aggregate emissions. Let Q^f denote the number of permits that the regulator allocates for free to borrowers. Using this notation, the complementary slackness condition of the borrower is equivalent to (D.6) with Q^f taking the place of ϕQ.

To isolate the effect of allocating permits for free, we aim to perform a comparative statics exercise in which the price of permits remains constant. That is, we evaluate the effect of increasing Q^f on the demand for permits by borrowers E^d at a given permit price p. We note that this implies the total supply of permits may need to adjust to keep p fixed. The effect of free permits on the demand for permits, for a given price p, is

$$\left.\frac{\partial E^d}{\partial Q^f}\right|_p = \frac{\partial E(X, I_1)}{\partial X}\left.\frac{\partial X^*}{\partial Q^f}\right|_p + \frac{\partial E(X, I_1)}{\partial I_1}\left.\frac{\partial I_1^*}{\partial Q^f}\right|_p. \tag{D.10}$$

We thus need to find $\partial X^*/\partial Q^f|_p$ and $\partial I_1/\partial Q^f|_p$. To do that notice that Eqs. (D.4) and (D.6) (where Q^f takes the place of ϕQ) respectively define $X^* = f(I_1, p)$ and $I_1^* = g(X, p, Q^f)$. This implies

$$\begin{aligned}\left.\frac{\partial X^*}{\partial Q^f}\right|_p &= \frac{\partial f}{\partial I_1}\frac{\partial I_1}{\partial Q^f} = \frac{\partial X^*}{\partial I_1}\frac{\partial I_1}{\partial Q^f}, \\ \left.\frac{\partial I_1^*}{\partial Q^f}\right|_p &= \frac{\partial g}{\partial X}\frac{\partial X}{\partial Q^f} + \frac{\partial g}{\partial Q^f} = \frac{\partial I_1^*}{\partial X}\frac{\partial X}{\partial Q^f} + \frac{\partial I_1^*}{\partial Q^f}.\end{aligned}$$

Next we differentiate (D.6) with respect to X to find $\partial I_1^*/\partial X$:

$$\tilde{r}(p)\frac{\partial I_1^*}{\partial X} - \left(p\frac{\partial E(X, I_1)}{\partial X} + \frac{\partial C(X, I_1)}{\partial X}\right) = 0 \;\Rightarrow\; \frac{\partial I_1^*}{\partial X} = 0.$$

where we use that FOC (D.4) implies $p\partial E(X, I_1)/\partial X + \partial C(X, I_1)/\partial X = 0$. Differentiating (D.6) in which Q^f takes the place of ϕQ, with respect to Q^f:

$$\tilde{r}(p)\frac{\partial I_1^*}{\partial Q^f} + p = 0 \;\Rightarrow\; \frac{\partial I_1^*}{\partial Q^f} = \frac{-p}{\tilde{r}(p)}.$$

Differentiating (D.4) with respect to I_1 to find $\partial X^*/\partial I_1$ yields:

$$\begin{aligned}&\left(p\frac{\partial^2 E(X, I_1)}{\partial X\partial I_1} + \frac{\partial^2 C(X, I_1)}{\partial X\partial I_1}\right) + \left(p\frac{\partial^2 E(X, I_1)}{\partial X^2} + \frac{\partial^2 C(X, I_1)}{\partial X^2}\right)\frac{\partial X^*}{\partial I_1} = 0 \\ &\Rightarrow \frac{\partial X^*}{\partial I_1} = -\frac{p\frac{\partial^2 E(X,I_1)}{\partial X\partial I_1} + \frac{\partial^2 C(X,I_1)}{\partial X\partial I_1}}{\frac{\partial^2 C(X,I_1)}{\partial X^2}} = \frac{\frac{\partial^2 N(p,X,I_1)}{\partial X\partial I_1}}{\frac{\partial^2 C(X,I_1)}{\partial X^2}}.\end{aligned}$$

Using these in (D.10) yields

$$\begin{aligned}\left.\frac{\partial E^d}{\partial Q^f}\right|_p &= \left(\frac{\partial E(X, I_1)}{\partial X}\frac{\partial X^*}{\partial I_1} + \frac{\partial E(X, I_1)}{\partial I_1}\right)\frac{\partial I_1^*}{\partial Q^f} \\ &= \left(\frac{\partial E(X, I_1)}{\partial X}\frac{\frac{\partial^2 N(p,X,I_1)}{\partial X\partial I_1}}{\frac{\partial^2 C(X,I_1)}{\partial X^2}} + \frac{\partial E(X, I_1)}{\partial I_1}\right)\frac{(-p)}{\tilde{r}(p)}.\end{aligned}$$

Since $\tilde{r}(p) < 0$, we get that $\partial E^d/\partial Q^f|_p < 0$ if and only if

$$\left(\frac{\partial E(X, I_1)}{\partial X}\frac{\frac{\partial^2 N(p,X,I_1)}{\partial X\partial I_1}}{\frac{\partial^2 C(X,I_1)}{\partial X^2}} + \frac{\partial E(X, I_1)}{\partial I_1}\right) < 0,$$

Plugging back in $\partial X^*/\partial I_1 = \left(\partial^2 N(p,X,I_1)/\partial X \partial I_1\right) \times \left(\partial^2 C(X,I_1)/\partial X^2\right)^{-1}$ yields the condition stated in Lemma 3.

Appendix E. Initial leverage choice

E.1. Changes to the setup

The key changes to the model are described in the main text in Section 5.1. Additionally, there is a $t = 0$ budget constraint and a revised $t = 1$ budget constraint given by, respectively,

$$c_0 + I_0 - d_0 = A^b, \tag{E.1}$$

$$c_1 + d_0 + C(X, I_1) = \mu(I_1 - I_0) + d_1, \tag{E.2}$$

where the borrowers' inside equity in the project is $e = I_0 - d_0$. We do not restrict d_0 to be positive, so it should be interpreted as net debt between $t = 0$ and $t = 1$.

E.2. Restated borrower problem

Throughout our analysis, we focus on the case in which the borrower finds it optimal to start the project at $t = 0$, rather than to forgo it and consume all of the endowment. We first show that the necessary condition for this is that the model parameters are such that the borrower's optimal initial debt satisfies $d_0^* < \mu I_0$. To see this, suppose that the borrower would like to borrow $d_0 > \mu I_0$ at $t = 0$. Since the borrower's financial slack at $t = 1$ decreases in I_1, by Assumption 2.2, the overall resources that are available to the borrower at $t = 1$ are maximized at $I_1 = I_0$ where they equal μI_0. Hence, a borrower with initial debt of $d_0 > \mu I_0$ would have to default. This, combined with the fact that investors can force liquidation of the assets and appropriate all liquidation proceeds, implies a $t = 0$ constraint on borrowing given by $d_0 \le \mu I_0$. Would borrowers want to borrow to the point where this constraint just binds, $d_0 = \mu I_0$? In this case, investors would force liquidation at $t = 1$ to recoup their initial debt because μI_0 is the highest pledgeable income. Thus, borrower utility would be given by $u(A^b - I_0(1-\mu))$. But this is dominated by forgoing the project and fully consuming the endowment at $t = 0$, which gives the borrower $u(A^b)$. Therefore, borrowers would always forgo the project if the optimal $d_0^* \ge \mu I_0$. This motivates our focus on parameter ranges in which the optimal initial debt is such that $d_0^* < \mu I_0$. In this range, the $t = 0$ borrowing constraint is always slack.

Thus, if the borrower finds it optimal to start the project, the modified version of the Lagrangian (A.2) in the presence of the ex-ante leverage choice is given by

$$\begin{aligned}\max_{X,I_1,d_1,e} \mathcal{L} =& u(A_0 - e) + \mu(I_0 - I_1) - C(X, I_1) + R(I_1, E^a) - \tau E(X, I_1) + T \\ &+ \lambda\left[\theta R(I_1, E^a) - \tau E(X, I_1) + \psi T - d_1\right] + \underline{\kappa}_I I_1 + \overline{\kappa}_I [I_0 - I_1] \\ &+ \kappa_{c_1}\left[d_1 + e - I_0 + \mu(I_0 - I_1) - C(X, I_1)\right].\end{aligned} \tag{E.3}$$

The first order conditions with respect to X and I_1 are the same as in the baseline model (8) and (9), as stated in the main text. Similarly, the first order condition w.r.t. d_1 implies that $\lambda = \kappa_{c_1}$ as in the baseline model. This implies we can again combine the complementary slackness condition of the financial constraint (5) and the non-negativity constraint for c_1 in (6), which results in the modified condition (17) stated in the main text.

E.3. Proof of Proposition 6

Consider the problem of a regulator who maximizes social welfare by choosing $\bar{e}$ at $t = 0$ and τ at $t = 1$. The first order condition with respect to τ is given by Eq. (11). This implies that the optimal tax is still characterized by Proposition 3, as stated in the main text.

The first order conditions of the regulator with respect to $\bar{e}$ is

$$\begin{aligned}&u'(A^b - \bar{e}) \\ &= 1 + r(\gamma, X^*, I_1^*)\frac{\partial I_1^*}{\partial \bar{e}} - \left(\gamma \frac{\partial E(X, I_1)}{\partial X} + \frac{\partial C(X, I_1)}{\partial X}\right)\frac{\partial X^*}{\partial \bar{e}}\end{aligned} \tag{E.4}$$

To get Eq. (19) in the main text, combine this FOC with the borrower's FOC w.r.t. X, (8), and totally differentiate (8) with respect to $\bar{e}$ to find:

$$\frac{\partial X^*}{\partial \bar{e}} = \frac{\partial X^*}{\partial I_1}\frac{\partial I_1^*}{\partial \bar{e}}. \tag{E.5}$$

Effect of equity on liquidations and abatement. To derive the result in Proposition 6, we first find $\frac{\partial I_1^*}{\partial \bar{e}}$ and $\frac{\partial X^*}{\partial I_1^*}$, which are needed to further expand Eq. (E.4).

Totally differentiating (8) with respect to I_1 allows us to find

$$\frac{\partial X^*}{\partial I_1} = \frac{\frac{\partial^2 N(X, I_1, \tau)}{\partial X \partial I_1}}{\frac{\partial^2 C(X, I_1)}{\partial X^2}} \tag{E.6}$$

where we use $N(X, I_1, \tau) = -\tau E(X, I_1) - C(X, I_1)$.

If $\lambda^*(\tau) = 0$, then $I_1^* = I_0$, so $\partial I_1^*/\partial \bar{e} = 0$ and $\partial X^*/\partial \bar{e} = 0$. If $\lambda^*(\tau) > 0$, the interior solution of $I_1^*(\tau)$ is pinned down by (10). Totally differentiating (10) with respect to $\bar{e}$ yields:

$$\frac{\partial I_1^*}{\partial \bar{e}} = \frac{-1 - (\psi\tau - \theta\gamma^p)\frac{\partial E(X, I_1)}{\partial X}\frac{\partial X^*}{\partial I_1}\frac{\partial I_1^*}{\partial \bar{e}}}{\tilde{r}(\tau(1-\psi) + \theta\gamma^p, X^*, I_1^*)} \tag{E.7}$$

Combining (E.5) and (E.7) and using the shorthand notation, yields:

$$\frac{\partial I_1^*}{\partial \bar{e}} = \frac{-C''_{X^2}}{\tilde{r}(\tau(1-\psi) + \theta\gamma^p, X^*, I_1^*)C''_{X^2} + (\psi\tau - \theta\gamma^p)E'_X N''_{XI}} \tag{E.8}$$

$$\frac{\partial X^*}{\partial \bar{e}} = \frac{-N''_{XI}}{\tilde{r}(\tau(1-\psi) + \theta\gamma^p, X^*, I_1^*)C''_{X^2} + (\psi\tau - \theta\gamma^p)E'_X N''_{XI}} \tag{E.9}$$

The denominators of (E.8) and (E.9) are negative by Assumption 3 in Appendix B.1. The numerator of (E.8) is negative since $C''_{X^2} > 0$ by Assumption 1.1. This implies that $\frac{\partial I_1^*}{\partial \bar{e}} > 0$.

From (E.8), $\partial X^*/\partial \bar{e} > 0$ if and only if the cross-derivative $N''_{XI} = \partial^2 N(X, I_1, \tau)/\partial X \partial I > 0$, with $N(X, I_1, \tau)$ defined in Eq. (A.1). The economic interpretation of this cross-derivative being positive is that the net benefit of abatement is greater at a higher investment scale, for example, because there are economies of scale which make it cheaper to reduce emissions of a larger project. However, the effect of higher equity can alternatively be negative if $N''_{XI} < 0$.

Comparing private and socially optimal equity choice. Focusing on the case where the financial constraint binds and using (9), (E.8), and (E.9), we can further rewrite the regulator's FOC (19) and the borrower's FOC (18) as, respectively,

$$u'(A^b - \bar{e}) - 1 = \frac{-r(\tau, X^*, I_1^*)C''_{X^2} + (\gamma - \tau)[E'_I C''_{X^2} + E'_X N''_{XI}]}{\tilde{r}(\tau(1-\psi) + \theta\gamma^p, X^*, I_1^*)C''_{X^2} + (\psi\tau - \theta\gamma^p)E'_X N''_{XI}}, \tag{E.10}$$

$$u'(A^b - e) - 1 = \frac{-r(\tau, X^*, I_1^*)}{\tilde{r}(\tau, X^*, I_1^*)}. \tag{E.11}$$

Comparing the two, borrowers choose an inefficiently low level of equity if and only if:

$$\frac{-r(\tau, X^*, I_1^*)C''_{X^2} + (\gamma - \tau)[E'_I C''_{X^2} + E'_X N''_{XI}]}{\tilde{r}(\tau(1-\psi) + \theta\gamma^p, X^*, I_1^*)C''_{X^2} + (\psi\tau - \theta\gamma^p)E'_X N''_{XI}} > \frac{-r(\tau, X^*, I_1^*)}{\tilde{r}(\tau, X^*, I_1^*)} \tag{E.12}$$

Note that under Assumption 3 stated in Appendix B.1:

$$\tilde{r}(\tau(1-\psi) + \theta\gamma^p, X^*, I_1^*)C''_{X^2} + (\psi\tau - \theta\gamma^p)E'_X N''_{XI} < 0,$$

and by Assumption 2: $\tilde{r}(\tau, X^*, I_1^*) < 0$. Thus with some algebra, (E.12) becomes:

$$\left(\frac{\partial E(X, I_1)}{\partial I_1} + \frac{\partial E(X, I_1)}{\partial X} \frac{\frac{\partial^2 N(X, I_1, \tau)}{\partial X \partial I_1}}{\frac{\partial^2 C(X, I_1)}{\partial X^2}} \right) \times \left[(\gamma - \tau) - \frac{r(\tau, X^*, I_1^*)}{\tilde{r}(\tau, X^*, I_1^*)} (\theta \gamma^p - \psi \tau) \right] < 0. \tag{E.13}$$

To further simplify we use (E.6) to restate the first term of (E.13) as:

$$\frac{\partial E(X, I_1)}{\partial I_1} + \frac{\partial E(X, I_1)}{\partial X} \frac{\frac{\partial^2 N(X, I_1, \tau)}{\partial X \partial I_1}}{\frac{\partial^2 C(X, I_1)}{\partial X^2}} = \frac{\partial E(X, I_1)}{\partial I_1} + \frac{\partial E(X, I_1)}{\partial X} \frac{\partial X^*}{\partial I_1} \equiv \frac{dE}{dI_1}$$

Since $\partial I_1^*/\partial \bar{e} > 0$ whenever $\lambda^* > 0$, we can express the condition in terms of $dE(X^*, I_1^*)/d\bar{e}$, by using $dE(X^*, I_1^*)/d\bar{e} = dE(X^*, I_1^*)/dI_1 \times \partial I_1^*/\partial \bar{e}$. Plugging in, the following condition characterizes whether borrowers choose a socially optimal level of equity

$$\frac{dE(X^*, I_1^*)}{d\bar{e}} \underbrace{\left[\gamma - \tau^* + \lambda \left(\theta \gamma^p - \psi \tau^* \right) \right]}_{\text{T-SCC wedge}} = 0. \tag{E.14}$$

Borrowers choose a lower level of equity than the regulator if the left-hand side is smaller than zero. Conversely, borrowers choose a higher level of equity if the left-hand side is greater than zero, and the same level if it is equal to zero.

The left-hand side of Eq. (E.14) measures the gap in the marginal social and private values of increasing financial slack by contributing more equity. It consists of the marginal effect of equity on emissions, $dE(X^*, I_1^*)/d\bar{e}$, and a *total social cost of carbon* (T-SCC) wedge. The T-SCC wedge reflects the difference between the direct social and private cost of emissions, $\gamma - \tau$, as well as the effect of emissions on pledgeable income due to collateral externality associated with physical climate impacts, and tax rebates, $\lambda (\theta \gamma^p - \psi \tau)$. From Proposition 4, the optimal emissions tax is equal to τ^{GP} if $\psi = 1$, which implies a zero T-SCC wedge and no motive for leverage regulation. By contrast, if $\psi < 1$, the optimal emissions tax is below the generalized Pigouvian benchmark, so that the T-SCC wedge is positive and leverage regulation can improve welfare, as stated in Proposition 6.

E.4. The effect of equity on emissions

This subsection characterizes under what conditions equity has a positive or negative effect on emissions. If the financial constraint is slack, then $dE(X^*, I_1^*)/d\bar{e} = 0$. To understand how leverage affects emissions when the financial constraint binds, note that:

$$\frac{dE(X^*, I_1^*)}{d\bar{e}} = \underbrace{\left(\underbrace{\frac{\partial E(X, I_1)}{\partial I_1}}_{\text{Direct effect of } I_1} + \underbrace{\frac{\partial E(X, I_1)}{\partial X} \frac{\partial X^*}{\partial I_1}}_{\text{Indirect effect through } X^*} \right)}_{= \frac{dE}{dI_1}} \frac{\partial I_1^*}{\partial \bar{e}}. \tag{E.15}$$

Higher borrower equity relaxes financial constraints, which allows the borrower to liquidate less, and therefore implies a higher final investment scale, $\partial I_1^*/\partial \bar{e} > 0$. The direct effect of a higher investment scale is an increase in emissions, captured by the first term in brackets in Eq. (E.15). At the same time, looser financial constraints affect the optimal abatement choice. This effect is captured by the second term in brackets in Eq. (E.15). Note that this is an indirect effect that depends on how the marginal cost and benefit of abatement respond to changes in the final investment scale. As reflected in Eq. (E.6) the magnitude and direction of this effect depends on the cross-derivatives of $C(X, I_1)$ and $E(X, I_1)$, since $\partial^2 N(X, I_1, \tau)/\partial X \partial I_1 = -\tau \partial^2 E(X, I_1)/\partial X \partial I_1 - \partial^2 C(X, I_1)/\partial X \partial I_1$:

- The effect of higher equity on abatement is positive if abatement is more efficient at a higher investment scale: i.e. if $\partial^2 N(X, I_1, \tau)/\partial X \partial I_1 > 0$.
- The effect of higher equity on abatement is negative if abatement is less efficient at a higher investment scale: i.e. if $\partial^2 N(X, I_1, \tau)/\partial X \partial I_1 < 0$.

Since both the direct and indirect effects operate through the final investment scale, the overall sign of $dE/d\bar{e}$ coincides with the sign of dE/dI_1. Using this insight, we can combine Eqs. (19) and (E.15) to parsimoniously describe the optimal leverage mandate.

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